8.3 CI for μ, σ NOT known (old 8.4)

GOALS:
1. Learn the properties of the *student-t* distribution and the *t-curve*.
2. Understand how degrees of freedom, *df*, relate to *t-curves.*
3. Recognize that t-curves approach the SNC as *df* increases.
4. Perform the *t-interval* procedure to find the confidence interval when σ is not known.

Study 8.3,# 109-119,123-127(75-91*), 133(~97*)
*old 8.4

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8.3 CI for μ, σ NOT known (old 8.4)

In more realistic situations, σ is NOT known.

Need to use sample *s* instead of σ

But, can NOT use standardized version of $\bar{X}$
ie: no z score

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8.3 CI for μ, σ NOT known (old 8.4)

Wikipedia: Why is this procedure called the ***student t-test ?***

The *t*-statistic was introduced in 1908 by William Sealy Gosset, a chemist working for the Guinness brewery in Dublin, Ireland. "Student" was his pen name.[1][2][3][4]

Gosset had been hired owing to Claude Guinness's policy of recruiting the best graduates from Oxford and Cambridge to apply biochemistry and statistics to Guinness's industrial processes.[2] Gosset devised the *t*-test as an economical way to monitor the quality of stout. The *t*-test work was submitted to and accepted in the journal *Biometrika* and published in 1908.[5] Company policy at Guinness forbade its chemists from publishing their findings, so Gosset published his statistical work under the pseudonym "Student" (see Student's *t*-distribution for a detailed history of this pseudonym, which is not to be confused with the literal term *student*).

σ

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8.3 CI for μ, σ NOT known (old 8.4)

SNC	*t* - curve
standardized $\bar{x}$	studentized $\bar{x}$
$z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$	$t = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$
n.d. or large *n*	n.d. or large *n*
σ known	σ **not** known
same for all n	different curve for each *n*

estimating

1 variable: $\bar{x}$ 2 variables: $\bar{x}$ s

σ

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8.3 CI for μ, σ NOT known (old 8.4)

SNC	*t* - curve
only 1 variable	2 variables
$\bar{x}$	$\bar{x}$ s

When σ is known, there is only one parameter to estimate, the population *μ*. Therefore, there is only one variable, $\bar{x}$

SNC	*t* - curve
standardized $\bar{x}$	studentized $\bar{x}$
$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$	$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$
n.d. or large *n*	n.d. or large *n*
σ known	σ not known
same for all n	different curve for each *n*

When σ is NOT known, there are two parameters to estimate: the population *μ,* and the population standard deviation, σ. Therefore, there are 2 variables, $\bar{x}$ and s

Many t-curves- different curve for each n.
When you change n, change $\sigma_{\bar{x}}$ and change shape.

8.3 CI for μ, σ NOT known (old 8.4)

calculator demo **(not required)**:

1. y_1= normalpdf(x,0,1)
2. y_2= tpdf(x,1)
3. use stat to enter 2, 4, 20 into L1
4. y_3= tpdf(x,L1)
5. y_4= tpdf(x,100)

window: -3<x<3, 0<y<0.4

not required

geogebra t-curve demo

df = n-1

For df =1, the t curve is wider and shorter than SNC. What does this tell us about the two curves?(Hint: shape)

As df increases, t- curves --> snc

8.3 CI for μ, σ NOT known (old 8.4)

Properties of ***t - curve***

1. Total area under curve = 1.
2. Approaches horizontal axis as asymptote
3. Symmetric about 0.
4. As the df increases, ***t***-curves --> SNC

} same as the SNC

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8.3 CI for μ, σ NOT known (old 8.4)

Degrees Of Freedom

The number of values in a study that are free to vary.

eg: If have 5 pieces of fruit in a bowl, and you eat one each day.

On Day 1, you have a choice of	5
Day 2	4
Day 3	3
Day 4	2
Day 5	NO choice

For n = 5, free to choose 4 times:

df = n - 1 = 4

Degrees of Freedom refers to the maximum number of logically independent values, which are values that have the **freedom to vary**, in the data sample.

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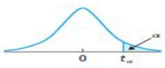
8.3 CI for μ, σ NOT known (old 8.4)

Classical approach, **now can use calculator**

TABLE IV
Values of t_α



only has five areas in the tail: 0.10, 0.05, 0.025, 0.01, 0.005; as subscripts

df	$t_{0.10}$	$t_{0.05}$	$t_{0.025}$	$t_{0.01}$	$t_{0.005}$	df
1	3.078	6.314	12.706	31.821	63.657	1
2	1.886	2.920	4.303	6.965	9.925	2
3	1.638	2.353	3.182	4.541	5.841	3
4	1.533	2.132	2.776	3.747	4.604	4
5	1.476	2.015	2.571	3.365	4.032	5
6	1.440	1.943	2.447	3.143	3.707	6
7	1.415	1.895	2.365	2.998	3.499	7
8	1.397	1.860	2.306	2.896	3.355	8
9	1.383	1.833	2.262	2.821	3.250	9
10	1.372	1.812	2.228	2.764	3.169	10
11	1.363	1.796	2.201	2.718	3.106	11
12	1.356	1.782	2.179	2.681	3.055	12
13	1.350	1.771	2.160	2.650	3.012	13
14	1.345	1.761	2.145	2.624	2.977	14
15	1.341	1.753	2.131	2.602	2.947	15
16	1.337	1.746	2.120	2.583	2.921	16
17	1.333	1.740	2.110	2.567	2.898	17
18	1.330	1.734	2.101	2.552	2.878	18
19	1.328	1.729	2.093	2.539	2.861	19
20	1.325	1.725	2.086	2.528	2.845	20
21	1.323	1.721	2.080	2.518	2.831	21
22	1.321	1.717	2.074	2.508	2.819	22
23	1.319	1.714	2.069	2.500	2.807	23
24	1.318	1.711	2.064	2.492	2.797	24
25	1.316	1.708	2.060	2.485	2.787	25
26	1.315	1.706	2.056	2.479	2.779	26
27	1.314	1.703	2.052	2.473	2.771	27
28	1.313	1.701	2.048	2.467	2.763	28
29	1.311	1.699	2.045	2.462	2.756	29
30	1.310	1.697	2.042	2.457	2.750	30
31	1.309	1.696	2.040	2.453	2.744	31
32	1.309	1.694	2.037	2.449	2.738	32
33	1.308	1.692	2.035	2.445	2.733	33
34	1.307	1.691	2.032	2.441	2.728	34
35	1.306	1.690	2.030	2.438	2.724	35
36	1.306	1.688	2.028	2.434	2.719	36
37	1.305	1.687	2.026	2.431	2.715	37
38	1.304	1.686	2.024	2.429	2.712	38
39	1.304	1.685	2.023	2.426	2.708	39
40	1.303	1.684	2.021	2.423	2.704	40
41	1.303	1.683	2.020	2.421	2.701	41
42	1.302	1.682	2.018	2.418	2.698	42
43	1.302	1.681	2.017	2.416	2.695	43
44	1.301	1.680	2.015	2.414	2.692	44
45	1.301	1.679	2.014	2.412	2.690	45
46	1.300	1.679	2.013	2.410	2.687	46
47	1.300	1.678	2.012	2.408	2.685	47
48	1.299	1.677	2.011	2.407	2.682	48

TABLE IV (cont.)
Values of t_α

df	$t_{0.10}$	$t_{0.05}$	$t_{0.025}$	$t_{0.01}$	$t_{0.005}$	d
	.299	1.676	2.009	2.403	2.678	
	.298	1.675	2.008	2.402	2.676	
	.298	1.675	2.007	2.400	2.674	
	.298	1.674	2.006	2.399	2.672	
	.297	1.674	2.005	2.397	2.670	
	.297	1.673	2.004	2.396	2.668	
	.297	1.673	2.003	2.395	2.667	
	.297	1.672	2.002	2.394	2.665	
	.296	1.672	2.002	2.392	2.663	
	.296	1.671	2.001	2.391	2.662	
	.296	1.671	2.000	2.390	2.660	
	.296	1.670	2.000	2.389	2.659	
	.295	1.670	1.999	2.388	2.657	
	.295	1.669	1.998	2.387	2.656	
	.295	1.669	1.998	2.386	2.655	
	.295	1.669	1.997	2.385	2.654	
	.295	1.668	1.997	2.384	2.652	
	.294	1.668	1.996	2.383	2.651	
	.294	1.668	1.995	2.382	2.650	
	.294	1.667	1.995	2.382	2.649	
	.294	1.667	1.994	2.381	2.648	
	.294	1.667	1.994	2.380	2.647	
	.293	1.666	1.993	2.379	2.646	
	.293	1.666	1.993	2.379	2.645	
	.293	1.666	1.993	2.378	2.644	
	.293	1.665	1.992	2.377	2.643	
	.292	1.664	1.990	2.374	2.639	
	.292	1.663	1.988	2.371	2.635	
	.291	1.662	1.987	2.368	2.632	
	.291	1.661	1.985	2.366	2.629	
	.290	1.660	1.984	2.364	2.626	1
	.286	1.653	1.972	2.345	2.601	2
	.284	1.650	1.968	2.339	2.592	3
	.284	1.649	1.966	2.336	2.588	4
	.283	1.648	1.965	2.334	2.586	5
	.283	1.647	1.964	2.333	2.584	6
	.283	1.647	1.963	2.332	2.583	7
	.283	1.647	1.963	2.331	2.582	8
	.282	1.647	1.963	2.330	2.581	9
	.282	1.646	1.962	2.330	2.581	10
	.282	1.646	1.961	2.328	2.578	20
	.282	1.645	1.960	2.326	2.576	
	$z_{0.10}$	$z_{0.05}$	$z_{0.025}$	$z_{0.01}$	$z_{0.005}$	

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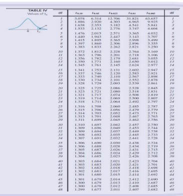
Class Notes

8.3 CI for μ, σ NOT known (old 8.4)

Table IV (back page of book)
t-values for different α values
and df.



G: df = 17
F: a) $t_{0.05}$ b) $t_{0.025}$ c) $t_{0.005}$

use TI84 calculator:
2nd VARS (distr)
4: invT(area, df)

TI83 calculator:
NO invT(area, df)
use: online stat book

Skip to CI. Values of $t_{\alpha/2}$ needed when using Table IV and formula.

-

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Class Notes

Homework

8.3 CI for μ, σ NOT known (old 8.4)

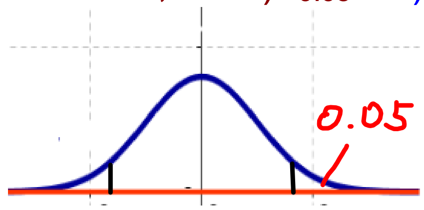
Skip to CI. Values of $t_{\alpha/2}$ needed when using Table IV and formula.

Table IV (back page of book) t-values for different α values and df.

TI84 calculator:
2nd VARS (distr)
4: invT(area, df)

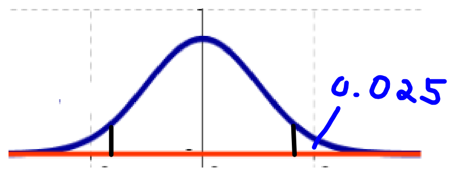
G: **df = 17**, F: a) $t_{0.05}$ b) $t_{0.025}$ c) $t_{0.005}$

a)



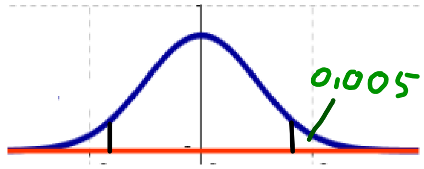
invT(0.05,17)=-1.7396
by symmetry: $t_{0.05}$=1.740
3 decimal digits

b)



invT(0.025,17)= -2.1098
by symmetry: $t_{0.025}$= 2.110

c)



invT(0.005,17)= -2.8982
by symmetry: $t_{0.005}$= 2.898

TI83 calculator:
NO invT(area, df)
use: online stat book

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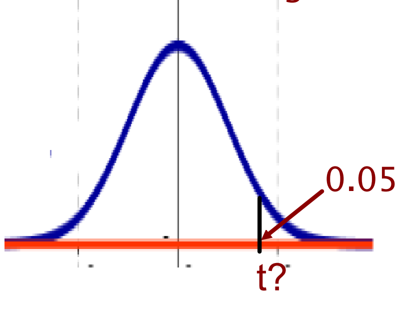
8.3 CI for μ, σ NOT known (old 8.4)

Skip to CI. Values of $t_{\alpha/2}$ needed when using Table IV and formula.

G: df = 8,
F: a) t ∋ 0.05 to right b) $t_{0.10}$
c) t ∋ 0.01 to left d) 2 t values ∋ center 0.95

a) t ∋ 0.05 to right



invT(area in tail,df)

a) invT(0.05,8) =−1.8595
symmetry: t = 1.860

TI83 calculator:
NO invT(area, df)
use: online stat book

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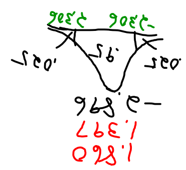


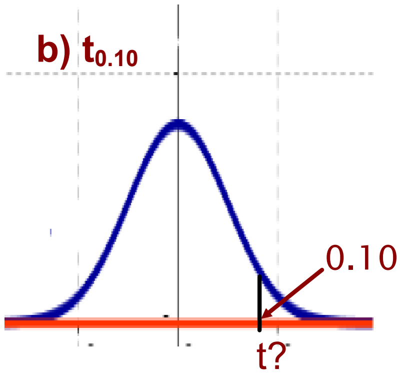
8.3 CI for μ, σ NOT known (old 8.4)

Skip to CI. Values of $t_{\alpha/2}$ needed when using Table IV and formula.

G: df = 8,

F: a) t ∋ 0.05 to right b) $t_{0.10}$

c) t ∋ 0.01 to left d) 2 t values ∋ center 0.95



invT(area in tail,df)

b) invT(0.10,8) =−1.3968

symmetry: t = 1.397

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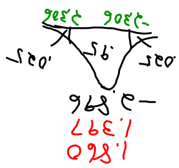


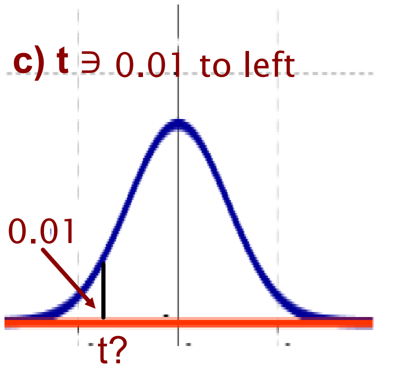
8.3 CI for μ, σ NOT known (old 8.4)

Skip to CI. Values of $t_{\alpha/2}$ needed when using Table IV and formula.

G: df = 8,

F: a) t ∋ 0.05 to right b) $t_{0.10}$

c) t ∋ 0.01 to left d) 2 t values ∋ center 0.95



invT(area in tail,df)

c) invT(0.01,8) =−2.8964

t = −2.896

TI83 calculator:
NO invT(area, df)
use: online stat book

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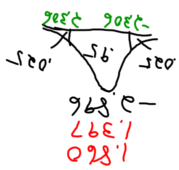


8.3 CI for μ, σ NOT known (old 8.4)

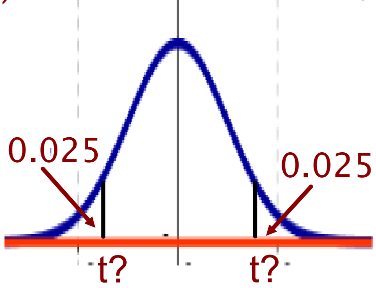
Skip to CI. Values of $t_{\alpha/2}$ needed when using Table IV and formula.

G: df = 8,

F: a) t ∋ 0.05 to right b) $t_{0.10}$

c) t ∋ 0.01 to left d) 2 t values ∋ center 0.95

d) 2 t values ∋ center 0.95



invT(area in tail,df)

center 0.95 ---> α=0.05 in tails, or
α/2= 0.025 in each tail

d) invT(0.025,8) =−2.3060

t = ±2.306

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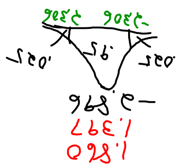

8.3 CI for μ, σ NOT known (old 8.4)

☆ Find a CI for μ: *t* - interval ☆

Assumptions: 1. Simple Random Sample
2. nd or large n
3. σ **un**known

Procedure

1. For CL of 1- α find $t_{\alpha/2}$ from Table IV or **invT(area, df)**
   df = n - 1, where n = sample size
2. Find CI:

$$\bar{x} - t_{\alpha/2}\cdot\frac{s}{\sqrt{n}} \le \mu \le \bar{x} + t_{\alpha/2}\cdot\frac{s}{\sqrt{n}}$$

3. Intepret:
   a) If n.d., CI precise
   b) If not n.d., n large, CI approximate

We will use the t-interval procedure on the calculator. So, do not need to specify the $t_{\alpha/2}$ value

TI83 calculator:
NO invT(area, df)
use: online stat book

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8.3 CI for μ, σ NOT known (old 8.4)

☆ Find a CI for μ: *t* - interval ☆

Assumptions: 1. Simple Random Sample
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Procedure

1. For CL of 1- α find $t_{\alpha/2}$ from Table IV or **invT(area, df)**
   df = n - 1; where n = sample size
2. Find CI:

$$\bar{x} - t_{\alpha/2} \cdot \frac{s}{\sqrt{n}} \le \mu \le \bar{x} + t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

Standard Error

3. Intepret:
   a) If n.d., CI precise
   b) If not n.d., n large, CI approximate

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8.3 CI for μ, σ NOT known (old 8.4)

☆ Find a CI for μ: *t* - interval ☆

Assumptions: 1. Simple Random Sample
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3. σ **un**known

Procedure

1. For CL of 1- α find $t_{\alpha/2}$ from Table IV or **invT(area, df)** df = n - 1, where n = sample size
2. Find CI:

$$\bar{x} - t_{\alpha/2} \cdot \frac{s}{\sqrt{n}} \le \mu \le \bar{x} + t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

3. Intepret:
   a) If n.d., CI precise
   b) If not n.d., n large, CI approximate

REQUIRED:
1. Check assumptions
2. **sketch showing both CL and α/2
3. Write formula and df
4. Substitute into equation showing subscript on t as value of α / 2
5. STAT/TESTS/TInterval
6. Result as an interval
   _____ ≤ μ ≤ _____
7. Interpretation

Word Problem

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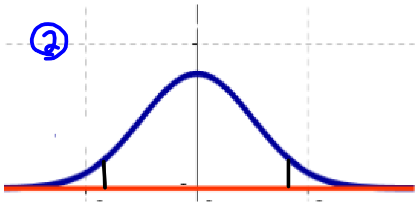
8.3 CI for μ, σ NOT known (old 8.4)

① srs, n large, σ not known

G: srs, $\bar{x} = 25$, n=36, s=3, CL=95% F: CI

②



CL = ____

α = 1 - ____ = ____

α/2 = ____

$t_{\alpha/2} = t$ ☐

df = n - 1 = ____

$$\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

REQUIRED:
1. Check assumptions
2. **sketch showing both CL and α/2
3. Write formula and df
4. Substitute into equation showing subscript on t as value of α / 2
5. STAT/TESTS/TInterval
6. Result as an interval
   ____ ≤ μ ≤ ____
7. Interpretation

calculator:
STAT / TESTS
8: TInterval
Inpt: STAT
x̄: 25
Sx: 3
n: 36
C-Level: .95
Calculate
Result: (23.985,26.015)

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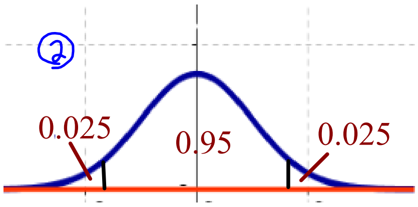
8.3 CI for μ, σ NOT known (old 8.4)

① srs, n large, σ not known

G: srs, $\bar{x} = 25$, n=36, s=3, CL=95% F: CI

②



CL = 0.95

α = 1 - 0.95 = 0.05

α/2 = 0.025

$t_{\alpha/2} = t_{0.025}$

df = n - 1 = 35

③ $$\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

④ $$25 \pm t_{0.025} \frac{3}{\sqrt{36}}$$

⑤ calculator:
STAT / TESTS
8: TInterval
Inpt: STAT
x̄ : 25
S_x: 3
n: 36
C-Level: .95
Calculate

Result: (23.985,26.015)

⑥ $23.99 \le \mu \le 26.02$

REQUIRED:
1. Check assumptions
2. **sketch showing both CL and α/2
3. Write formula and df
4. Substitute into equation showing subscript on t as value of α / 2
5. STAT/TESTS/TInterval
6. Result as an interval
   ____ ≤ μ ≤ ____
7. Interpretation

⑦ Conclude: We have 95% confidence that the population mean lies within the interval from 23.99 to 26.02

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8.3 CI for μ, σ NOT known (old 8.4)

Calculator details

calculator:
STAT / TESTS
8: TInterval
Inpt: STAT <- Could be DATA
$\bar{x}$: 25
S_x: 3
n: 36
C-Level: .95
Calculate

Result: (23.985,26.015)

If want to find $t_{0.025}$
TI84 calculator:
2nd VARS (distr)
4: invT(area, df)
But $t_{0.025}$ not actually required

TI83 calculator:
NO invT(area, df)
use: online stat book

$\bar{x}$

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8.3 CI for μ, σ NOT known (old 8.4)

G: A random sample of 16 batteries resulted in a mean weight of 55 gm and a standard deviation of 5 gm. If the weight is known to be normally distributed, estimate the population mean with a 99% confidence interval.

G: ______________________________ F: _______

REQUIRED:
1. Check assumptions
2. **sketch showing both CL and α/2
3. Write formula and df
4. Substitute into equation showing subscript on t as value of $\alpha / 2$
5. STAT/TESTS/TInterval
6. Result as an interval
_____ $\leq \mu \leq$ _____
7. Interpretation

$51.32 \leq \mu \leq 58.68$

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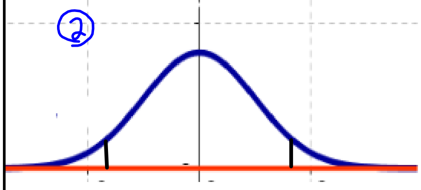
$\bar{x}$

8.3 CI for μ, σ NOT known (old 8.4)

G: A random sample of 16 batteries resulted in a mean weight of 55 gm and a standard deviation of 5 gm. If the weight is known to be normally distributed, estimate the population mean with a 99% confidence interval.

G: srs, nd, $\overline{x}$ = 55, n=16, s=5, CL=99% F: CI

① srs, nd, σ not known

② 

CL =_____

α = 1 - _____ = _____

α/2 = ____

$t_{\alpha/2}$ = t ☐

df = n -1 =______

$\overline{x}$

③ $\overline{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$

REQUIRED:
1. Check assumptions
2. **sketch showing both CL and α/2
3. Write formula and df
4. Substitute into equation showing subscript on t as value of α / 2
5. STAT/TESTS/TInterval
6. Result as an interval
_____ ≤ μ ≤ _____
7. Interpretation

51.32 ≤ μ ≤ 58.68

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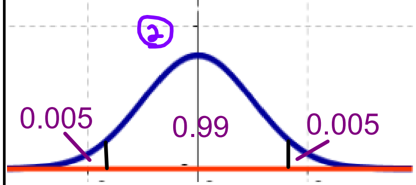
8.3 CI for μ, σ NOT known (old 8.4)

G: A random sample of 16 batteries resulted in a mean weight of 55 gm and a standard deviation of 5 gm. If the weight is known to be normally distributed, estimate the population mean with a 99% confidence interval.

G: srs, nd, $\overline{x}$ = 55, n=16, s=5, CL=99% F: CI

① Assumptions met: 1)srs 2)nd 3)σ not known

② 

CL = 0.99

α = 1 - 0.99 = 0.01

α/2 = 0.005

$t_{\alpha/2}$ = $t_{0.005}$

df = n -1 = 15

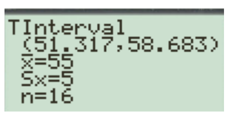
③ $\overline{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$

④ $55 \pm t_{0.005} \frac{5}{\sqrt{16}}$

⑥ 51.3mg ≤ μ ≤ 58.7mg

⑦ Conclude: Have 99% confidence that the population mean battery weight lies between 51.2 mg and 58.7 mg.

REQUIRED:
1. Check assumptions
2. **sketch showing both CL and α/2
3. Write formula and df
4. Substitute into equation showing subscript on t as value of α / 2
5. STAT/TESTS/TInterval
6. Result as an interval
_____ ≤ μ ≤ _____
7. Interpretation

⑤ 

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51.32 ≤ μ ≤ 58.68

p.383 8.136 The following data represent the age (in weeks) at which babies first crawl based on a randomized survey of 12 mothers.

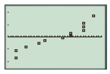
52	30	44	35
47	37	56	26
52	47	52	26

Find the 95% confidence interval for the mean number of weeks to a baby's first crawl.

REQUIRED:
1. Check assumptions
2. **sketch showing both CL and α/2
3. Write formula and df
4. Substitute into equation showing subscript on t as value of $\alpha / 2$
5. STAT/TESTS/TInterval
6. Result as an interval ____ ≤ μ ≤ ____
7. Interpretation

Switch steps 4 and 5 if have data.

Numeric results:
35.1 ≤ μ ≤ 48.9 weeks
x bar=42, s=10.79



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