8.2 Confidence Interval: one μ, σ known

GOALS:

1. Understand that a Confidence Interval
   a) estimates an interval of values within which the population mean, μ, lies
   b) is associated with a confidence **level**, which relates to area under the normal curve
2. **Perform the z-interval procedure** to find the Confidence Interval for μ when the population standard deviation, σ, is known.
3. Understand that the formula used for the CI actually refers to the distribution of $\bar{x}$, not the distribution of the population itself.

Study 8.2, # 27(13), 33(19), 37(21), 41(23), 63(25), 65(27), 67(29), 69(31)

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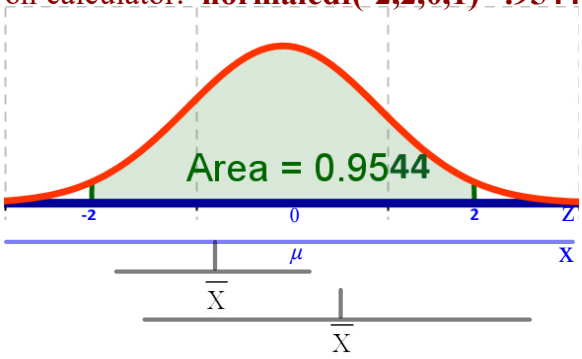
8.2 Confidence Interval: one μ, σ known

For the SNC, 95.44% of values are between z = -2 and z = 2

on calculator: **normalcdf(-2,2,0,1) .9544----**



For a sample from a normally distributed population, where is the sample mean relative to μ ?

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8.2 Confidence Interval: one μ, σ known

☆ Find a CI for μ ☆

Assumptions: 1. Simple Random Sample
2. nd or large n
3. σ known

If not n.d., and n is small, then can NOT use this procedure.

Procedure

1. For CL of 1- α find $z_{\alpha/2}$ from Table II

α is significance level
1 - α is confidence level

2. Find CI:

$$\bar{x} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

3. Intepret:
   a) If n.d., CI precise
   b) If not n.d., n large, CI approximate

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8.2 Confidence Interval: one μ, σ known

☆ Find a CI for μ ☆

Assumptions: 1. Simple Random Sample
2. nd or large n
3. σ known

Procedure

1. For CL of 1- α find $z_{\alpha/2}$ from Table II

2. Find CI:

$$\bar{x} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

3. Intepret:
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   b) If not n.d., n large, CI approximate

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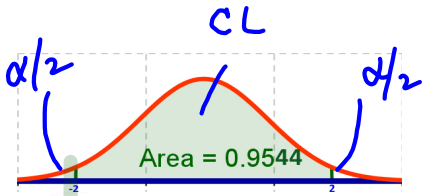
Find a CI for 95.44% confidence:



CL=.9544
α = 1-.9544
= 0.0456

8.2 Confidence Interval: one μ, σ known

★ Find a CI for μ ★

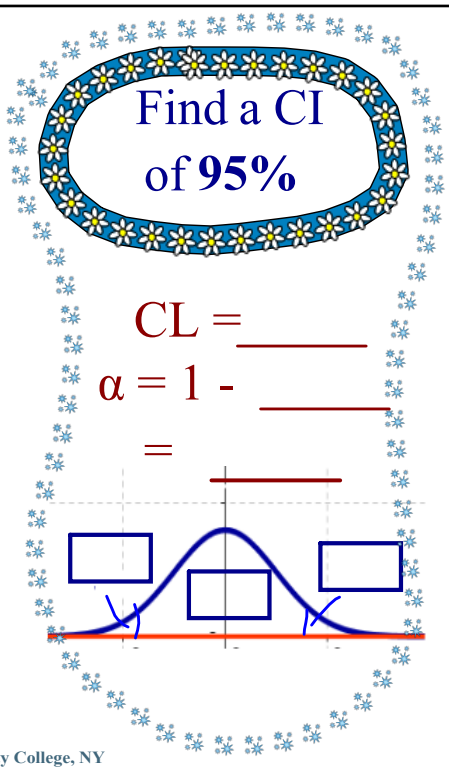
Assumptions: 1. Simple Random Sample
2. nd or large n
3. σ known

Procedure

1. For CL of 1- α find $z_{\alpha/2}$ from Table II
2. Find CI:

$$\bar{x} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

3. Intepret:
   a) If n.d., CI precise
   b) If not n.d., n large, CI approximate



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8.2 Confidence Interval: one μ, σ known

★ Find a CI for μ ★

Assumptions: 1. Simple Random Sample
2. nd or large n
3. σ known

Procedure

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3. Intepret:
   a) If n.d., CI precise
   b) If not n.d., n large, CI approximate

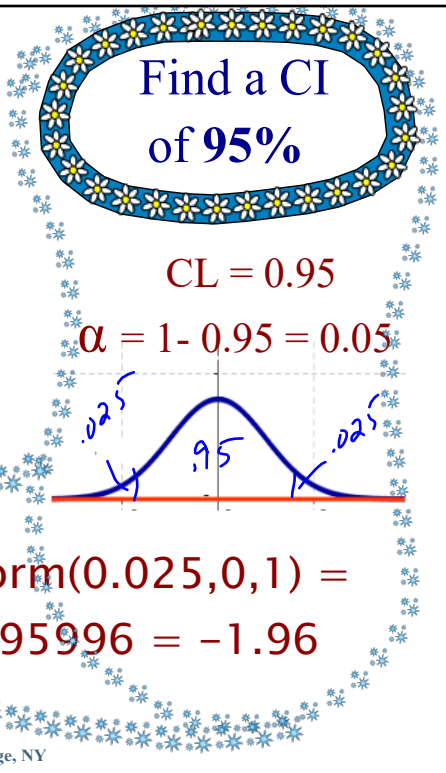

Table II

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8.2 Confidence Interval: one μ, σ known

★ Find a CI for μ ★

Assumptions: 1. Simple Random Sample
2. nd or large n
3. σ known

Procedure

1. For CL of 1- α find $z_{\alpha/2}$ from Table II
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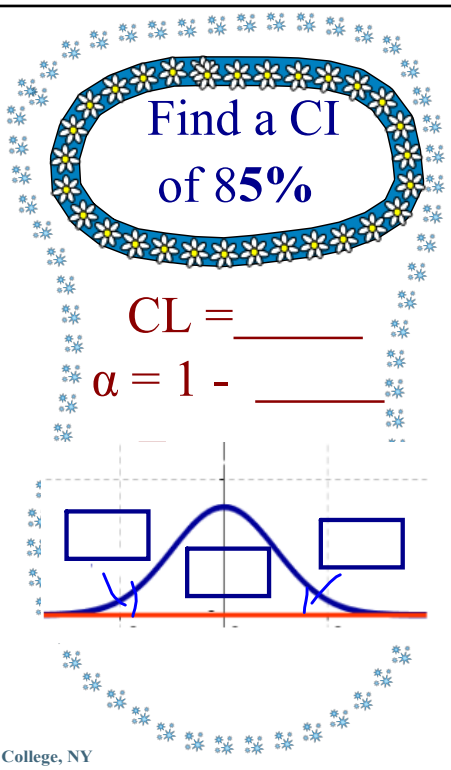
3. Intepret:
   a) If n.d., CI precise
   b) If not n.d., n large, CI approximate



invNorm(_____,0,1) =

= _____

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8.2 Confidence Interval: one μ, σ known

★ Find a CI for μ ★

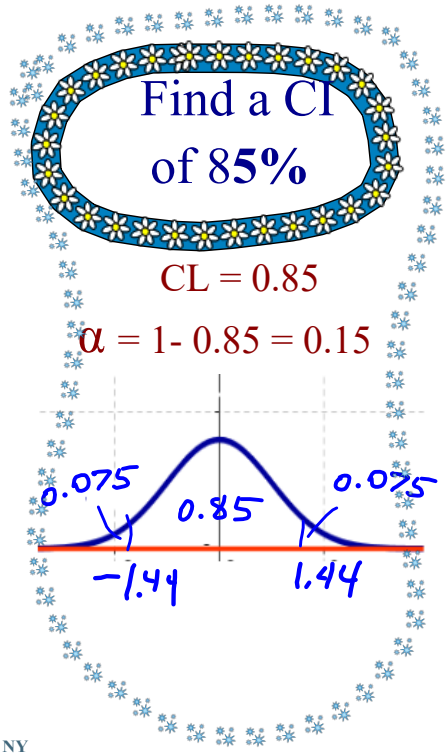
Assumptions: 1. Simple Random Sample
2. nd or large n
3. σ known

Procedure

1. For CL of 1- α find $z_{\alpha/2}$ from Table II
2. Find CI:

$$\bar{x} - 1.44 \cdot \frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + 1.44 \cdot \frac{\sigma}{\sqrt{n}}$$

3. Intepret:
   a) If n.d., CI precise
   b) If not n.d., n large, CI approximate



invNorm(0.075,0,1) =

-1.4395 = -1.44

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8.2 Confidence Interval: one μ, σ known

☆Find a CI for μ ☆

Assumptions: 1. Simple Random Sample
2. n.d. or large n
3. σ known

Procedure

1. For CL of 1- α find $z_{\alpha/2}$ from Table II
2. Find CI: $z_{\alpha/2} = z_{.025}$

$$\bar{x} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

3. Intepret:
   a) If n.d., CI precise
   b) If not n.d., n large, CI approximate

Some HW problems very simple:

#14 G: 95% CI
F: CL, α

8.2 Confidence Interval: one μ, σ known

☆Find a CI for μ ☆

Assumptions: 1. Simple Random Sample
2. n.d. or large n
3. σ known

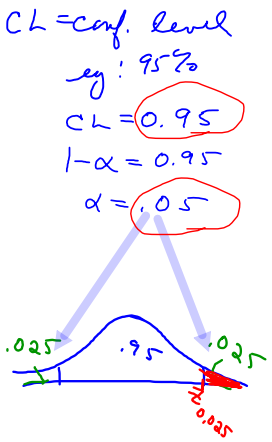
Procedure

1. For CL of 1- α find $z_{\alpha/2}$ from Table II
2. Find CI: $z_{\alpha/2} = z_{.025}$

$$\bar{x} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

3. Intepret:
   a) If n.d., CI precise
   b) If not n.d., n large, CI approximate

p.335 #14 G: 95% CI
F: CL, α



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8.2 Confidence Interval: one μ, σ known

Use caution in use of CI procedure when:

not n.d.
skewed
outliers

8.2 Confidence Interval: one μ, σ known

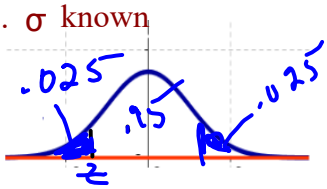
★Find a CI for μ★

Assumptions: 1. Simple Random Sample
2. n.d. or large n
3. σ known

G: s.r.s, $\bar{x}$ =25. n=36. σ = 3

F: 95% CI

Are the assumptions met?



CL = _____
α = 1 - _____ = _____
α/2 = _____
$z_{\alpha/2}$ = $z_{\square}$

$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

Procedure

1. For CL of 1- α find $z_{\alpha/2}$ from Table II
2. Find CI: $z_{\alpha/2} = z_{.025}$

$$\bar{x} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

3. Intepret:
   a) If n.d., CI precise
   b) If not n.d., n large, CI approximate

8.2 Confidence Interval: one μ, σ known

G: 1. s.r.s, $\bar{x}$ =25. 2. n=36. 3. σ = 3

F: 95% CI

Are the assumptions met? YES

CL = 0.95

α = 1 - 0.95 = 0.05

α/2 = 0.025

$z_{\alpha/2} = z_{0.025}$

$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

$25 \pm z_{0.025} \frac{3}{\sqrt{36}}$

(continue on next page)

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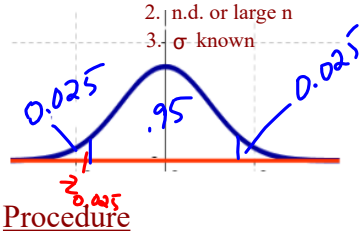
☆Find a CI for μ☆

Assumptions: 1. Simple Random Sample
2. n.d. or large n
3. σ known



Procedure

1. For CL of 1- α find $z_{\alpha/2}$ from Table
2. Find CI: $z_{\alpha/2} = z_{.025}$

$$\bar{x} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

3. Intepret:
   a) If n.d., CI precise
   b) If not n.d., n large, CI approximate

8.2 Confidence Interval: one μ, σ known

G: $\bar{x}$=25 n=36 σ=3 CL=95%

On calculator: STAT / TESTS / 7:ZInterval

then select or enter red below and select calculate

Inpt: Data Stats

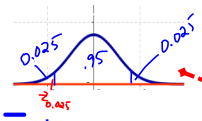
σ:3

$\bar{x}$: 25

n: 36

C-level: 0.95

Calculate



$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

$25 \pm z_{0.025} \frac{3}{\sqrt{36}}$

Output from Calculator:

(24.02,25.98)

the endpoints of the 95% CI

24.02 ≤ μ ≤ 25.98

Minimum requirement if done with calc:
1. Check assumptions
2. sketch showing both CL and α/2
3. Write formla
4. Substitute into equation showing subscript on z
5. STAT/TESTS/7:ZInterval
6. Result as an interval
   _____ ≤ μ ≤ _____
7. Interpretation

Conclude: We have 95% confidence that the population mean lies within the interval from 24.01 to 25.98

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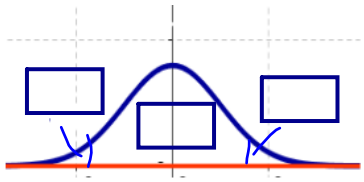
8.2 Confidence Interval: one μ, σ known

G: nd, srs, $\bar{x}$ =55, n=16, σ = 5

F: 99% CI



CL =_____

α = 1 - _____ = _____

α/2 = _____

$z_{\alpha/2}$ = $z_{\square}$

$\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$

☆Find a CI for μ ☆

Assumptions: 1. Simple Random Sample
2. n.d. or large n
3. σ known

Procedure

1. For CL of 1- α find $z_{\alpha/2}$ from Table II
2. Find CI:

$$\bar{x} - z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}$$

3. Intepret:
   a) If n.d., CI precise
   b) If not n.d., n large, CI approximate

Minimum requirement if done with calc:
1. Check assumptions
2. sketch showing both CL and α/2
3. Write formla
4. Substitute into equation showing subscript on z
5. STAT/TESTS/7:ZInterval
6. Result as an interval
   _____ ≤ μ ≤ _____
7. Interpretation

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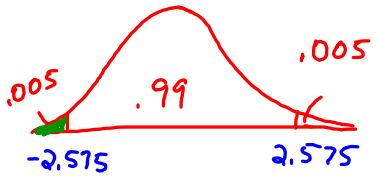
8.2 Confidence Interval: one μ, σ known

G: 2. nd, 1. srs, x =55, n=16, 3. σ = 5

Assumptions met.

F: 99% CI



α = 1-0.99 = 0.01

α/2 = .005

$\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 55 \pm z_{0.005}\frac{5}{\sqrt{16}} = 55 \pm 2.575\frac{5}{\sqrt{16}}$

51.78 < μ < 58.22

Procedure

1. For CL of 1- α find $z_{\alpha/2}$ from Table II
2. Find CI:

$$\bar{x} - z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}$$

3. Intepret:
   a) If n.d., CI precise
   b) If not n.d., n large, CI approximate

Conclude: We have 99% confidence that the population mean lies within the interval from 51.78 to 58.22

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8.2 Confidence Interval: one μ, σ known

CALCULATOR steps - note that the sketch, etc on previous page are required for a complete solution to this problem

G: $\bar{x}=55$, $n=16$, $\sigma=5$, $CL=99\%$

On calculator: STAT / TESTS / 7:ZInterval
then select or enter red below and select calculate

Inpt: Data Stats
σ: 5
$\bar{x}$: 55
n: 16
C-level: 0.99
Calculate

Use to check your answers using the z scores.

Output from Calculator:
(51.78,58.22)
the endpoints of the 99% CI

8.2 Confidence Interval: one μ, σ known

☆ Find a CI for μ ☆

Assumptions: 1. Simple Random Sample
2. nd or large n
3. σ known

Distribution of $\bar{x}$

Procedure

1. For CL of 1- α find $z_{\alpha/2}$ from Table II

$\sigma_{\bar{x}}$

2. Find CI:

$$\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

3. Intepret:
a) If n.d., CI precise
b) If not n.d., n large, CI approximate

Attachments

Statistical Tables.pdf