8.1 Estimating the Population Mean, μ

GOALS:

1. Understand that the sample mean is not expected to be exactly the same as the population mean.
2. Understand what a Point Estimate is.
3. Understand the differences between a statistic and a parameter.
4. Understand how a Confidence Interval improves the estimate of a population mean.

Study 8.1, # 1, 17(3), 19(5), 21(7), 23(9)

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8.1 Estimating the Population Mean, μ

If we don't know the value of the Population Mean, μ, what can we use to estimate it?

So far, best estimate for μ is the sample mean $\bar{x}$

Can we expect the sample mean to equal μ ?

μ σ

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7.1 Sampling Error

Distributions using DOTPLOTS

NY Yankees, 2011 Stats

	Player	Batting Average
C	Robinson Cano	302
J	Derek Jeter	297
R	Alex Rodriguez	276
G	Curtis Granderson	262
S	Nick Swisher	260

μ = 279.4
σ = 17.39

Samples of 2

Sample	Mean $\bar{x}$
CJ	299.5
CR	289.0
CG	282.0
CS	281.0
JR	286.5
JG	279.5
JS	278.5
RG	269.0
RS	268.0
GS	261.0

$\mu_{\bar{x}}=279.4$
$\sigma_{\bar{x}}=10.65$

Samples of 3

Sample	Mean $\bar{x}$
CJR	291.7
CJG	287.0
CJS	286.3
CRG	280.0
CRS	279.3
CGS	274.7
JRG	278.3
JRS	277.7
JGS	273.0
RGS	266.0

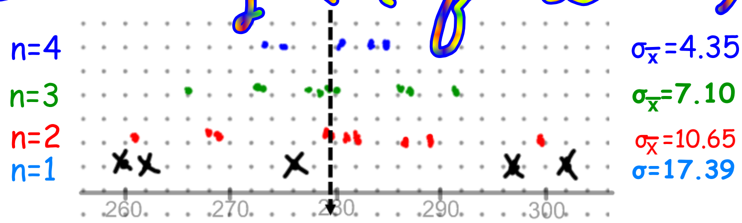
$\mu_{\bar{x}}=279.4$
$\sigma_{\bar{x}}=7.10$

Samples of 4

Sample	Mean $\bar{x}$
CJRG	284.25
CJRS	283.75
CJGS	280.25
CRGS	275.00
JRGS	273.75

$\mu_{\bar{x}}=279.4$
$\sigma_{\bar{x}}=4.35$

Is any $\bar{x}$ equal to μ?



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7.1 Sampling Error

Distributions using DOTPLOTS

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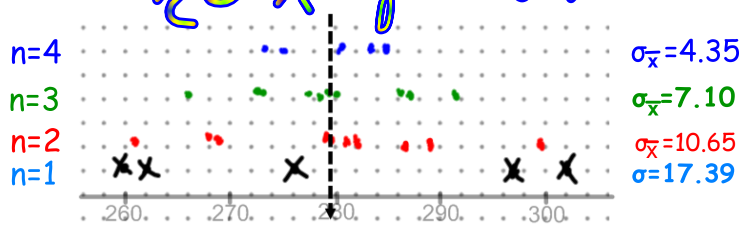
CJRS	283.75
CJGS	280.25
CRGS	275.00
JRGS	273.75

$\mu_{\bar{x}}=279.4$
$\sigma_{\bar{x}}=4.35$

no $\bar{x}$ equals μ



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8.1 Estimating the Population Mean, μ

The sample mean is not expected to equal the population mean.

Expect SAMPLING ERROR
(Difference between $\bar{x}$ and μ)

Goal:
Reduce amount of SAMPLING ERROR
Determine a measure of reliability associated with the estimate

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μ σ $\bar{x}$

8.1 Estimating the Population Mean, μ

Use sample mean, $\bar{x}$, to estimate the population mean μ

We call the sample mean a Point Estimate

A Point Estimate of a parameter is the value of the statistic used to estimate the parameter.

$\bar{x}$	statistic	variable
μ	parameter	fixed

Point Estimate	Parameter
$\bar{x}$	μ
s	σ
med	η

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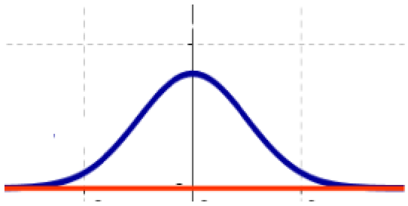
μ σ

$\bar{x}$

8.1 Estimating the Population Mean, μ

If $\overline{X} \neq \mu$
then it lies on either side
So, we look for an interval that contains μ,
using $\overline{X}$, known $\sigma_{\overline{X}}$, and snc

distribution of $\overline{X}$



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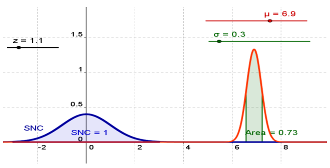
8.1 Estimating the Population Mean, μ

CONFIDENCE INTERVAL

An interval of numbers about a Point Estimate ($\overline{X}$) associated with a percent of confidence that the parameter lies within the interval.

Area under SNC between -z and +z



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7.1 Sampling Error

When sampling an unknown distribution, can use the sample mean to estimate the population mean.

C
J
R
G
S

μ = 279.4
σ = 17.39

Samples of 3

Sample	Mean $\bar{x}$
CJR	291.7
CJG	287.0
CJS	286.3
CRG	280.0
CRS	279.3
CGS	274.7
JRG	278.3
JRS	277.7
JGS	273.0
RGS	266.0

$$\sigma_{\bar{x}} = \sqrt{\frac{N-n}{N-1}}\,\frac{\sigma}{\sqrt{n}}$$

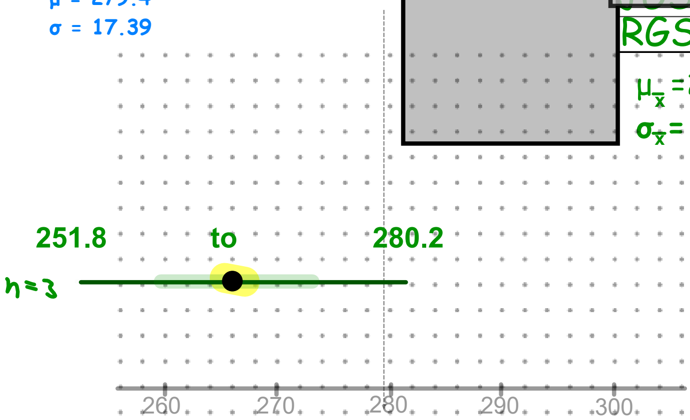
$$\sigma_{\bar{x}} = \sqrt{\frac{5-3}{5-1}}\;\frac{17.39}{\sqrt{3}}$$

$$= \sqrt{2/4}\cdot\frac{17.39}{\sqrt{3}}$$

$$= 7.10$$

$\mu_{\bar{x}} = 279.4$
$\sigma_{\bar{x}} = 7.10$

For 2 $\sigma_{\bar{x}}$ to right or left, use 14.20



If took a random sample of n=3 and found $\bar{x}$ = 266.0, then could consider an interval from 266.0 -14.20 to 266.0+14.20

Does this interval include μ, the mean of the population that we are trying to estimate?

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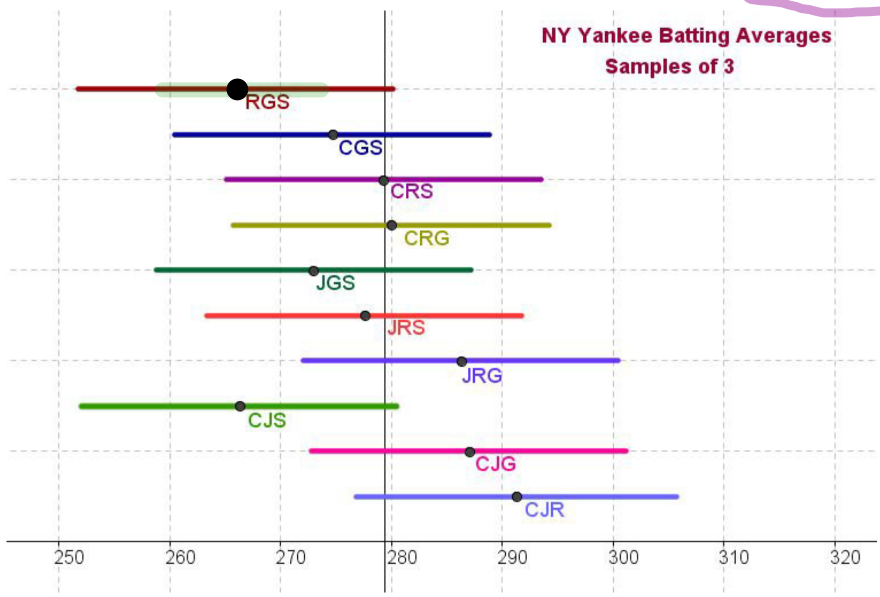
Statistics Home Page Class Notes Homework $\bar{x}$

8.1 Estimating the Population Mean, μ

95.44% Confidence Intervals

$$\bar{x} \pm 2\,\sigma_{\bar{x}}$$

95.44% because 0.9544 = area under SNC between -2σ and +2σ



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8.1 Estimating the Population Mean, μ

95.44% Confidence Intervals

$\bar{x} \pm 2\,\sigma_{\bar{x}}$

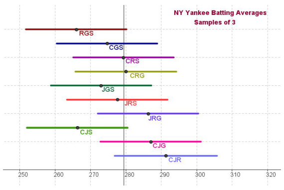
Estimating the population mean with:
1. sample mean and
2. known population std. dev.
3. sample size, n

Distribution of sample mean $\bar{x}$

$z = \frac{\bar{x}-\mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$

algebra

$$\bar{x} - z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}$$



$z_{\alpha/2}$ represents the z score for a particular significance level, α

For demonstration we used ± 2σ. Most often we use α that results in 95% or 99% confidence

SNC: $z = \frac{x-\mu}{\sigma}$

popul

$\bar{x}$

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8.1 Estimating the Population Mean, μ

95.44% Confidence Intervals

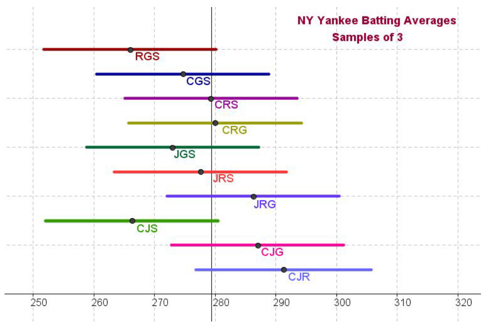
$\bar{x} \pm 2\,\sigma_{\bar{x}}$

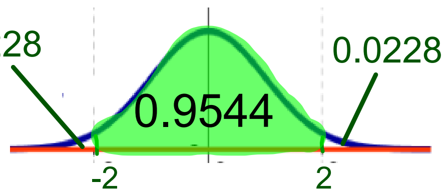
Estimating the population mean with:
1. sample mean and
2. known population std. dev.
3. sample size, n

α = 1 - 0.9544 = 0.0456

α / 2 = 0.0456/2 = 0.0228

$$\bar{x} - z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}$$





$z = \frac{x-\mu}{\sigma/\sqrt{n}}$

$z\frac{\sigma}{\sqrt{n}} = \bar{x}-\mu$

$\bar{x} \pm \boxed{z_{\alpha/2}}\boxed{\frac{\sigma}{\sqrt{n}}}$

$\sigma_{\bar{x}}$

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