6.1 Differential Equations & Slope Fields

HW: p. 407 # 1-5, 13, 15, 19-27,
37-47, 53, 55

6.1 Differential Equations & Slope Fields

Ch 3.9 Definition of Differentials:

Let y = f(x) be differentiable on an open interval containing x.

The **differential of *x*, *dx*,** is a non-zero real number.

The **differential of *y*, *dy*, is:** $\int dy = \int f'(x)\,dx$

Ch 4.1: Solve the differential equation:

$f'(x) = 4x, \qquad f(0) = 6$

$dy = f'(x)\,dx$

$dy = \frac{dy}{dx}\,dx$

$\int dy = \int 4x\,dx$

$y = \frac{4x^2}{2} + C = 2x^2 + C$

6.1 Differential Equations & Slope Fields

What about ....?

Solve the diff eq: $(2+x)\,y' - xy = 0$

$$\frac{dy}{dx} = \frac{xy}{x+2}$$

More complex than we have done so far.

This is where we are headed.

6.1 Differential Equations & Slope Fields

What about ....?

Solve the diff eq: $(2+x)\,y' - xy = 0$

Note that problems of this complexity are covered later in the chapter. This is offered here because students wanted to see that there is no new procedure involved.

$$\frac{dy}{dx} = \frac{xy}{x+2}$$

$$dy = \frac{dy}{dx}\,dx =$$

$$dy = \frac{xy}{x+2}\,dx$$

$$x+2 \;\overline{)\; x \;}\;\; 1 \qquad x+2 \qquad -2$$

$$\int \frac{dy}{y} = \int \frac{x}{x+2}\,dx = \int \left(1 - \frac{2}{x+2}\right) dx$$

$$\ln|y| = x - 2\int \frac{dx}{x+2}$$

$$\ln|y| = x - 2\ln|x+2| + c$$

$$e^{x-2\ln|x+2|+c} = y$$

$$e^x e^c e^{-2\ln|x+2|} = y$$

$$\frac{C_1 e^x}{(x+2)^2} = y$$

$$e^{-2\ln|x+2|} = \frac{1}{(x+2)^2}$$

$$e^{\ln(x+2)^{-2}} = \frac{1}{z^2}$$

$$\ln e^{\ln(x+2)^{-2}} = \ln z^{-2}$$

$$\ln(x+2)^{-2} \ln e = \ln z^{-2}$$

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Page

Homework Part 1

Title: solve diff eq (5 of 19)

6.1 Differential Equations & Slope Fields

Start with verify problems: Verify a solution is correct for the diff eq.

p. 409 # 2 G: $y = e^{-x}$; $3y' + 4y = e^{-x}$

$y' = -e^{-x}$

$3(-e^{-x}) + 4e^{-x} \stackrel{?}{=} e^{-x}$

$-3e^{-x} + 4e^{-x}$

$e^{-x} = e^{-x}$ ✓

6.1 Differential Equations & Slope Fields

4. $G: y^2 - 2\ln y = x^2$, $\frac{dy}{dx} = \frac{xy}{y^2-1}$

$2y\frac{dy}{dx} - 2\cdot\frac{1}{y}\frac{dy}{dx} = 2x$

$\left(y - \frac{1}{y}\right)\frac{dy}{dx} = x$

$\frac{dy}{dx} = \frac{x}{y-\frac{1}{y}}\cdot\frac{y}{y} = \frac{xy}{y^2-1}$ ✓

6.1 Differential Equations & Slope Fields

What about:

$$dy/dx = x + y$$

$$dy = \frac{dy}{dx}\,dx = (x+y)\,dx$$

If we cannot solve a diff eq analytically,

consider the slope field associated with the equation.

This gives us a graphical representation of the slopes of the general solution.

6.1 Differential Equations & Slope Fields

Links for Slope Fields

dy/dx represents the slope of y

Equation Plotter: Slope Fields

Start with: dy/dx = x

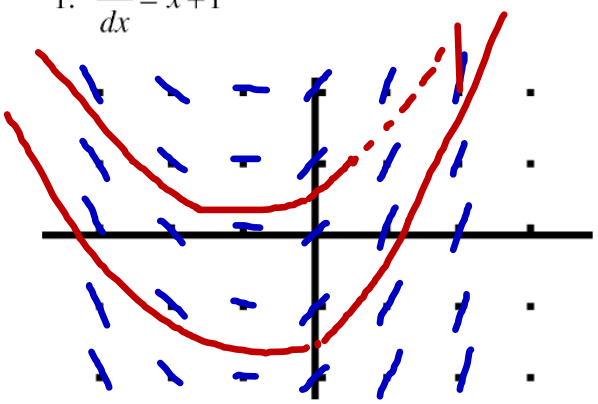
$$dy = \frac{dy}{dx}\,dx$$

$$\int dy = \int x\,dx$$

$$y = x^2 + C$$

Slope Field Worksheet
prepared by Nancy Stephenson
and available at apcentral.collegeboard.com

1. $\frac{dy}{dx} = x + 1$



6.1 Differential Equations & Slope Fields

410 #40

G: $\frac{dy}{dx} = \frac{e^x}{1+e^x}$ F: Solve

$dy = \frac{dy}{dx}\,dx$

$\int dy = \int \frac{e^x}{1+e^x}\,dx$

$u = 1+e^x$

$du = e^x dx$

$= \int \frac{du}{u} = \ln|u| + c = \ln|1+e^x| + C$

6.1 Differential Equations & Slope Fields

46. $\frac{dy}{dx} = x\sqrt{5-x}$ Solve.

$$\int dy = -\int x(5-x)^{1/2}\,dx$$

$u = 5-x$

$du = -dx$

$x = 5-u$

$$-\int (5-u)u^{1/2}\,du$$

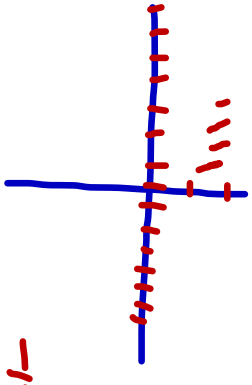
$$= -\int (5u^{1/2} - u^{3/2})\,du$$

6.1 Differential Equations & Slope Fields

54. $\frac{dy}{dx} = \frac{1}{2}\sin x$

$x = 0 \quad \frac{dy}{dx} = \frac{1}{2} \cdot 0 = 0$

$x = \frac{\pi}{2} \quad \frac{dy}{dx} = \frac{1}{2}\sin\frac{\pi}{2} = \frac{1}{2}$



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