5.3 Inverse Functions: Introduction

(Additional practice for 5.2: p. 338 #13, 15, 17, 41)

HW: 5.3: p. 347 #1, 5, 29, 33, 37, 41, 47, 51, 59, 77, 79, 23-25

Consider and compare:

$f(x) = -4x + 3$ and $g(x) = \frac{x - 3}{-4}$

Start with x. Then:

Note the inverse operations

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1-11, 10, 12, 13-19, 21, 23, 27,
31, 35, 41, 43, 59, 83, 85

5.3 Inverse Functions: Introduction

(Additional practice for 5.2: p. 338 #13, 15, 17, 41)

HW: 5.3: p. 347 #1, 5, 29, 33, 37, 41, 47, 51, 59, 77, 79, 23-25 7

Consider and compare:

$f(x) = -4x + 3$ and $g(x) = \frac{x - 3}{-4}$

Start with x. Then:

1. Mult by -4	1. Subt. 3
2. Add 3	2. Divide by -4

Note the inverse operations

5.3 Inverse Functions: Introduction

Consider and compare:

$f(x) = \sqrt{(2x - 3)}$ and $g(x) = \frac{x^2 + 3}{2}$

$x \geq 3/2$ $g \geq 3/2$

Start with x. Then:

These appear to be inverse functions, but what about x=0?
Are the inverse functions for all x? all y?

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5.3 Inverse Functions: Introduction

Consider and compare:

$f(x) = \sqrt{(2x - 3)}$ and $g(x) = \frac{x^2 + 3}{2}$

$x \geq 3/2$ $g \geq 3/2$

Start with x. Then:

1. Mult. by 2	1. Square
2. Subt. 3	2. Add 3
3. Sqrt.	3. Div. by 2

These appear to be inverse functions, but what about x=0?
Are the inverse functions for all x? all y?

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5.3 Inverse Functions: Introduction

How do you find $f^{-1}(x) = g(x)$?

$f(x) = \sqrt{(2x - 3)}$ and $g(x) = \frac{x^2 + 3}{2}$

$x \geq 3/2$ $g \geq 3/2$

1. rewrite f(x) as y
2. exchange x and y
3. solve for the new y

$f^{-1}(x)$

$y = \sqrt{2x-3}$

$x = \sqrt{2y-3}$

$x^2 = 2y - 3$

$x^2 + 3 = 2y$

$y = \frac{x^2+3}{2}$

These appear to be inverse functions, but what about x=0?
Are the inverse functions for all x? all y?

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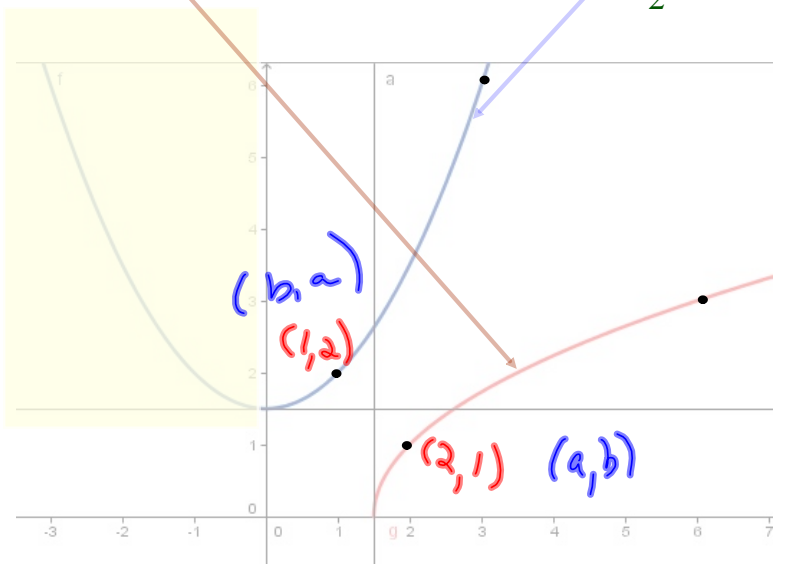
5.3 Inverse Functions: Introduction

$f(x) = \sqrt{(2x - 3)}$, $x \geq 3/2$ and $g(x) = \frac{x^2 + 3}{2}$, $x \geq 0$



But, *f* and *g* are inverses ONLY when $x \geq 0$ for *g(x)*

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Graph Preview

5.3 Inverse Functions: Definition

A function g is the inverse of a function f if

$f(g(x)) = x$ **for each** x **in the domain of** g

AND $g(f(x)) = x$ **for each** x **in the domain of** f

(f^{-1} notation means "f inverse", NOT exponent)

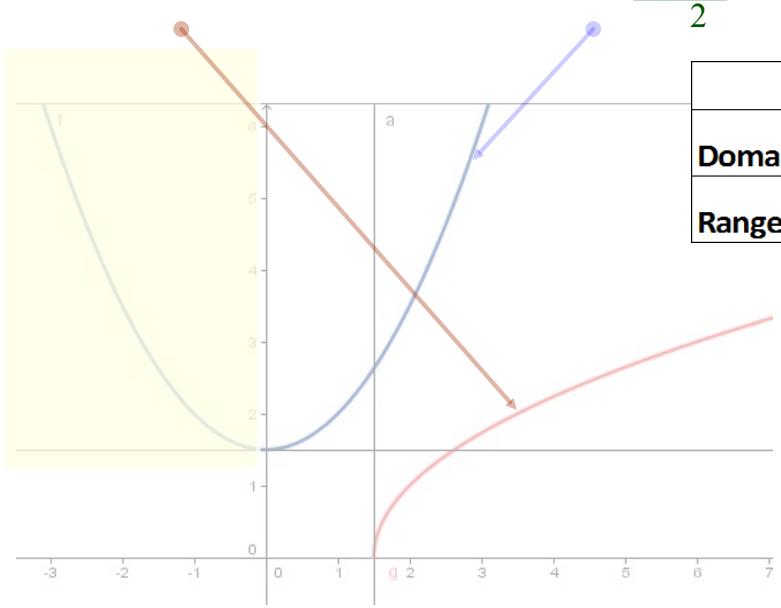
and $g(x) = f^{-1}(x)$

Notes

1. If $g(x) = f^{-1}(x)$, then $f(x) = g^{-1}(x)$
2. Domain of f^{-1} is the range of f and the range of f^{-1} is the domain of f
3. If f^{-1} exists, then it is unique (only one)
4. goes in both directions

5.3 Inverse Functions: Introduction

$f(x) = \sqrt{(2x - 3)}$, $x \geq 3/2$ and $g(x) = \frac{x^2 + 3}{2}$, $x \geq 0$



	f(x)	$f^{-1}(x)$
Domain		
Range		

But, *f* and *g* are inverses ONLY when $x \geq 0$ for *g(x)*

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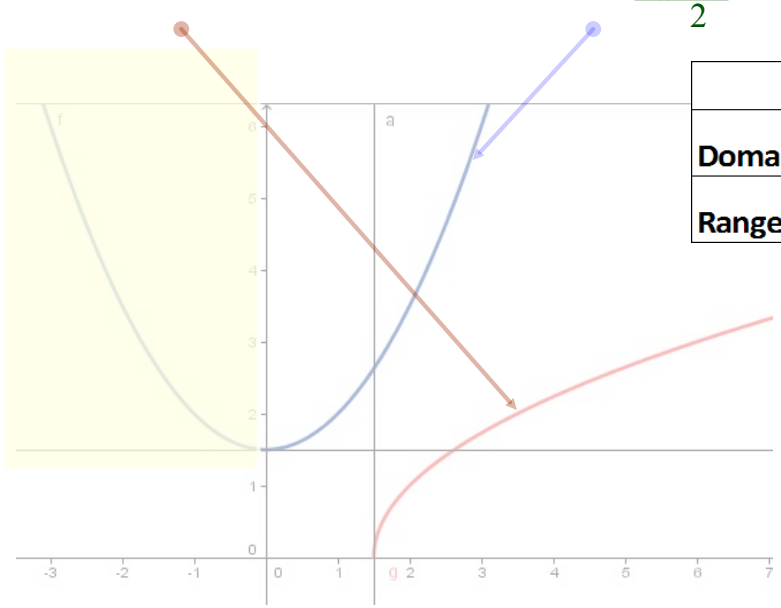
Graph Preview

5.3 Inverse Functions: Introduction

$f(x) = \sqrt{(2x - 3)}$, $x \geq 3/2$ and $g(x) = \frac{x^2 + 3}{2}$, $x \geq 0$



	f(x)	$f^{-1}(x)$
Domain	$x \geq 3/2$	$x \geq 0$
Range	$y \geq 0$	$y \geq 3/2$

But, *f* and *g* are inverses ONLY when $x \geq 0$ for *g(x)*

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Graph Preview

5.3 Inverse Functions

G: $f(x) = 3 - 4x$, $g(x) = \frac{3-x}{4}$ | F: Show $g(x) = f^{-1}(x)$

$\downarrow$

$f(g(x)) = x$

$g(f(x)) = x$

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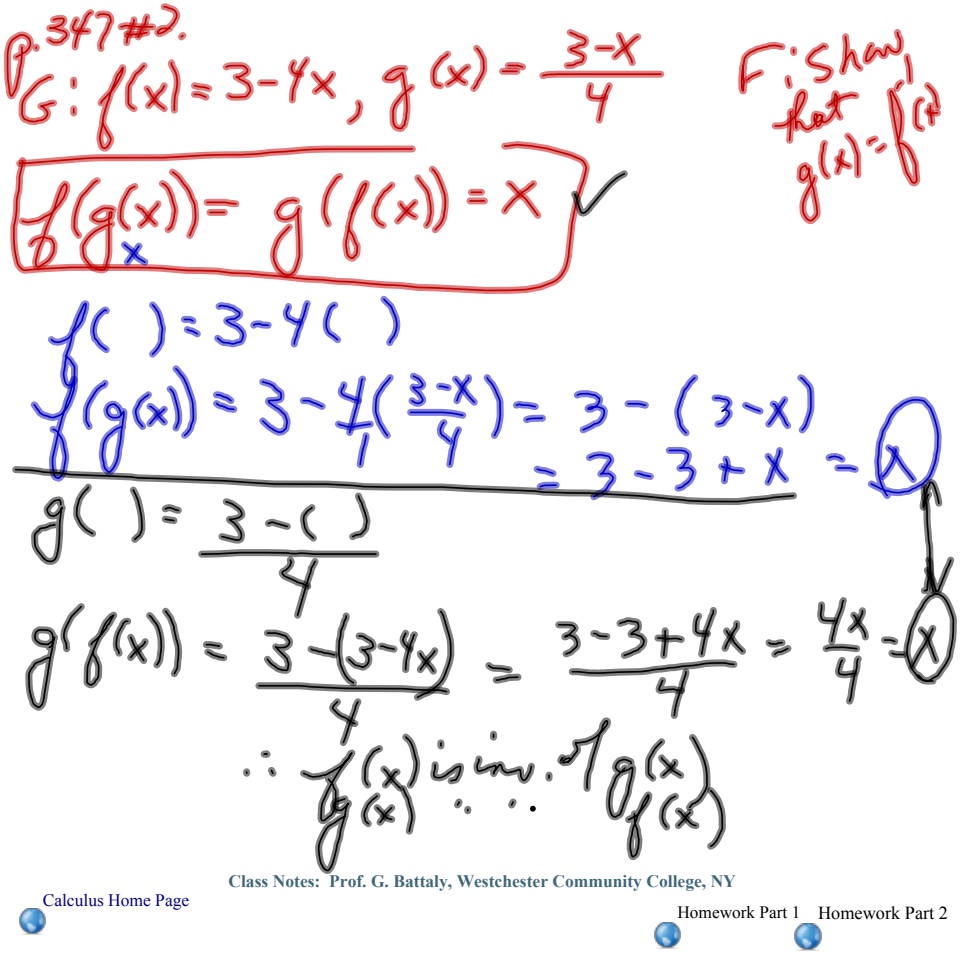
example

5.3 Inverse Functions



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example

5.3 Inverse Functions

5.3 #6

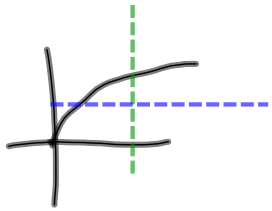
If a Function is 1 - to 1,

it has an inverse.

for each x ∃ exactly 1 y. **Vertical Line Test**

" " y ∃ " 1 x **Horizontal Line Tes**

eg: $y = \sqrt{x}$



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example

5.3 Inverse Functions

Explore relationships between a function and its inverse, using Geogebra. Click on the globe.
Uses a quadratic function.

Video of geogebra for lnx

$y = \ln x$

Explore relationships between a function and its inverse, using Geogebra. Click on the globe.
Uses *ln x*.

$y = e^x$

$\ln y = x$ → exchange variables for inverse → $y = \ln x$

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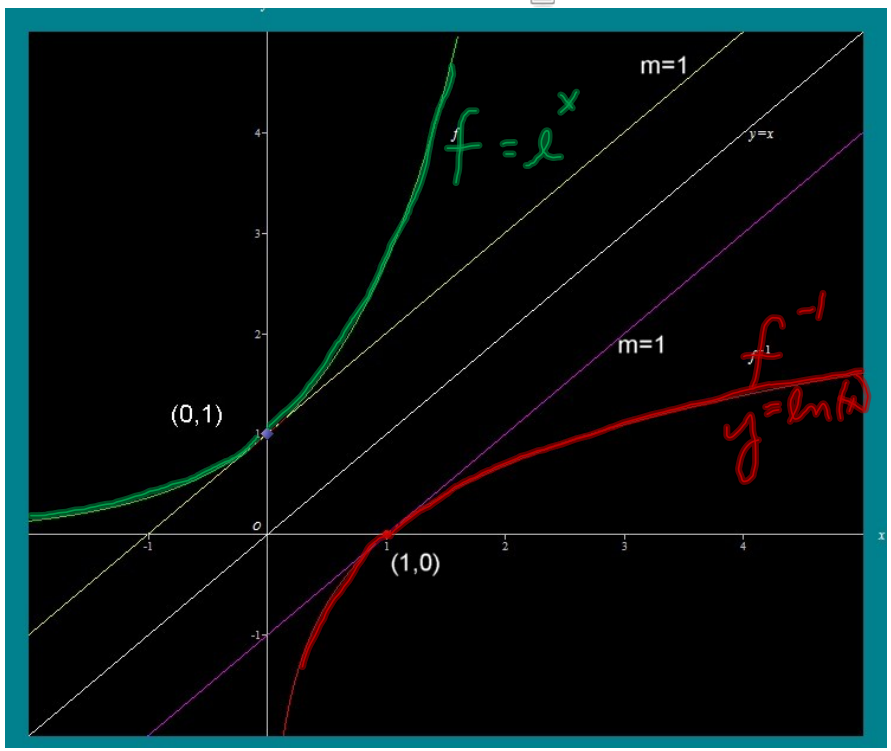
example

5.3 Inverse Functions: Properties

skip if have done geogebra for *ln x*



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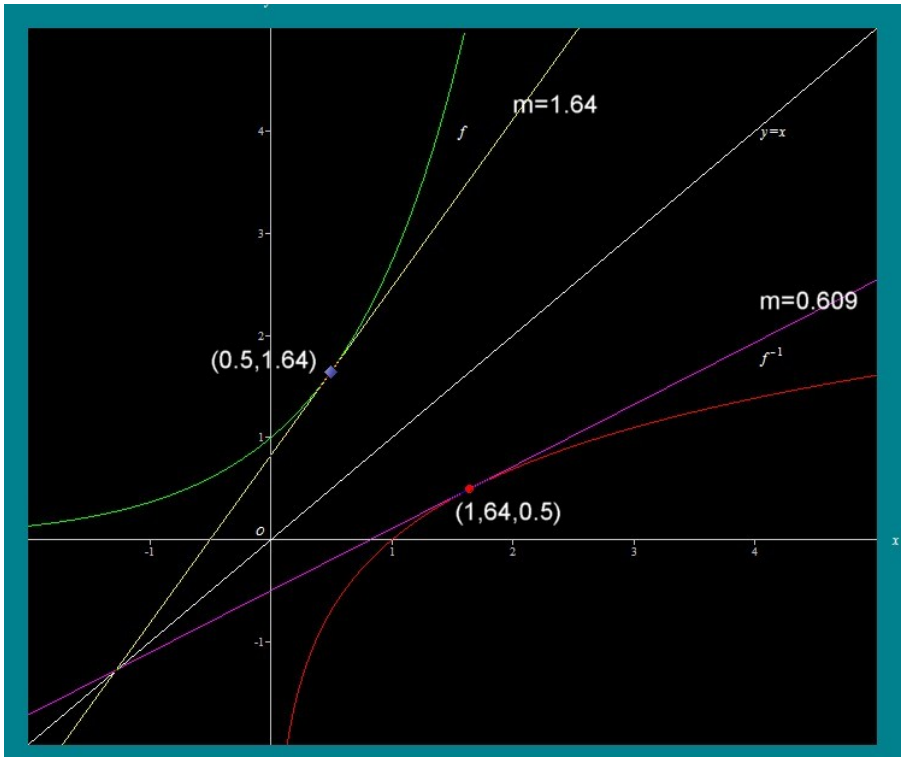
properties

5.3 Inverse Functions: Properties



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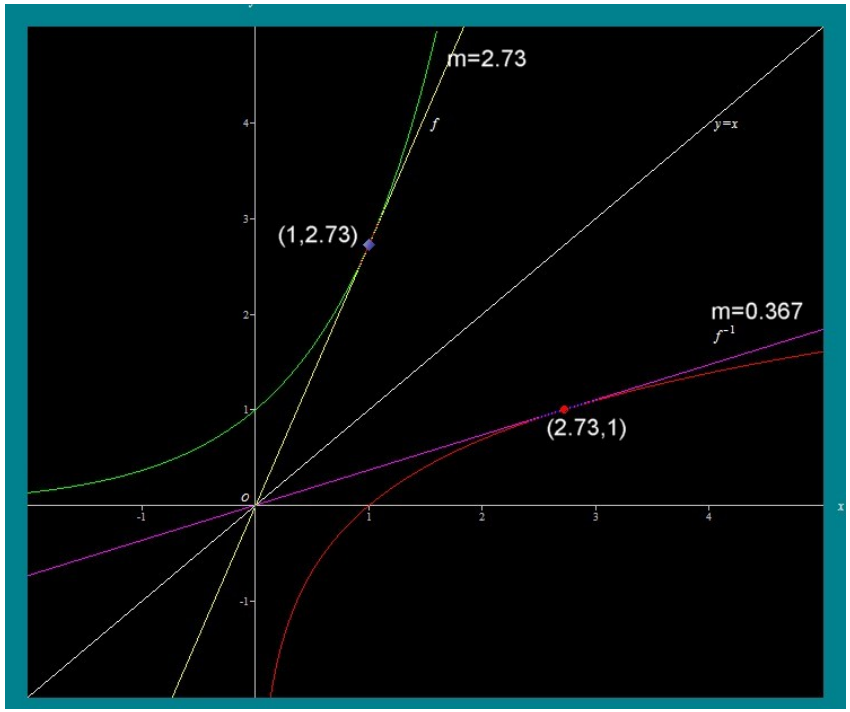
properties

5.3 Inverse Functions: Properties



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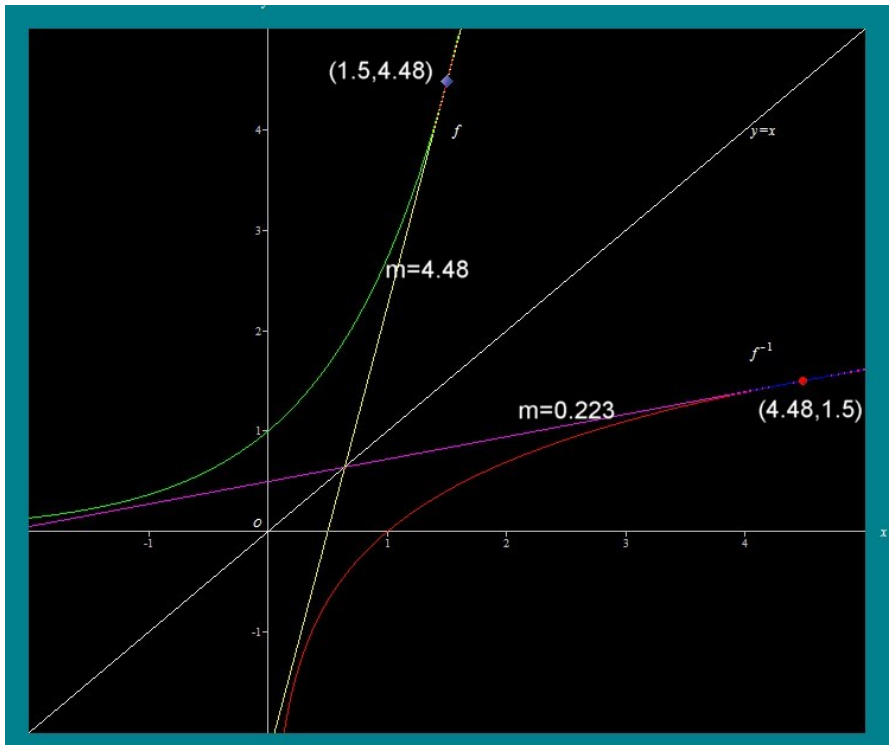
properties

5.3 Inverse Functions: Properties



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properties

5.3 Inverse Functions: Properties

(Insert images from APCD or demo the APCD)

(a,b) on f	slope of f	(b,a) on f^{-1}	slope of f^{-1}
(0,1)	1	(1,0)	1
	1.64		0.609
	2.73		0.369
	4.48		.223

(a,b) is on f

(b,a) on f^{-1}

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points and slopes

5.3 Inverse Functions:

Reflective Property of Inverse Function

The graph of f contains the point (a,b) IFF

the graph of f^{-1} contains the point (b,a)

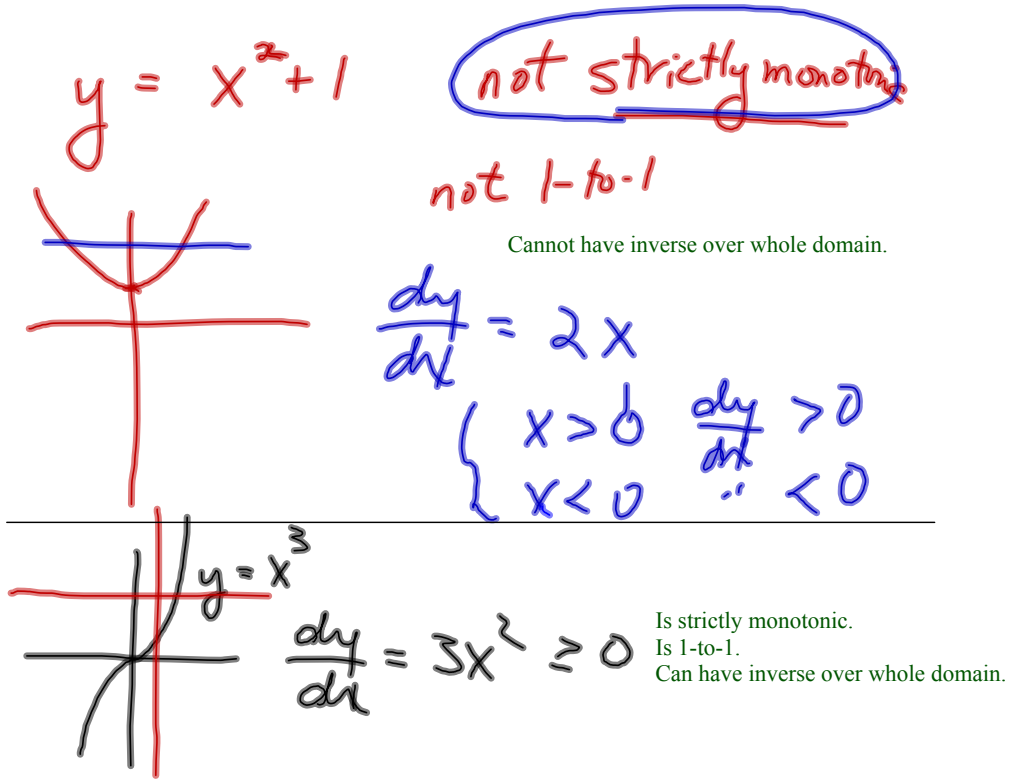
Derivative of an Inverse Function

Let f be a function differentiable on I, and $g(x) = f^{-1}(x)$

1. **Then g is differentiable at any x for which $f'(g(x)) \neq 0$**

2. $g'(x) = \dfrac{1}{f'(g(x))}$ **and** $f'(g(x)) \neq 0$

5.3 Inverse Functions:



Cannot have inverse over whole domain.

Is strictly monotonic.
Is 1-to-1.
Can have inverse over whole domain.

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Need 1-to-1

5.3 Inverse Functions:

$f(x) = 3x$ $F : f^{-1}$

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example

5.3 Inverse Functions:

$f(x) = 3x$ $\quad F: f^{-1}$

$y = 3x$

$x = 3y$ Exchange variables, and solve for y.

$y = \frac{x}{3} = f^{-1}(x) = g(x)$

$f(g(x)) = g(f(x)) = x$? Can check to be sure, using definition.

$f(\ ) = 3(\ )$

$f(g(x)) = 3\left(\frac{x}{3}\right) = x$

$g(f(x)) = x$? ✓

$g(\ ) = \frac{(\ )}{3}$

$g(f(x)) = \frac{3x}{3} = x$ ✓

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example

5.3 Inverse Functions:

$f(x) = \sqrt{x^2-4}, x \geq 2$ F: f^{-1}

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example

5.3 Inverse Functions:

$f(x) = \sqrt{x^2-4}$, $x \geq 2$ F: f^{-1}

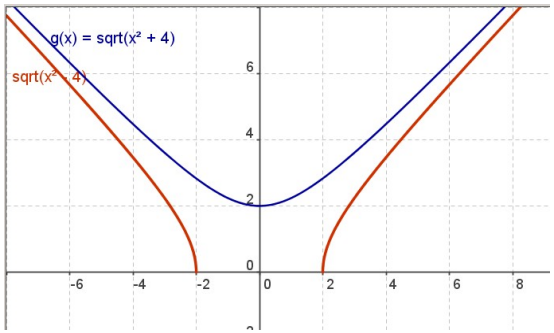
$y = \sqrt{x^2-4}$

$x = \sqrt{y^2-4}$

$x^2 = y^2 - 4$

$y^2 = x^2 + 4$

$f^{-1}: y = \sqrt{x^2+4}$



How do we restrict the domain for these to be inverse functions?

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example

5.3 Inverse Functions:

G: $f(x) = x^3 - 6x^2 + 12x$

F: Is f strictly monotonic? Use f ' (x)

5.3 Inverse Functions:

G: $f(x) = x^3 - 6x^2 + 12x$

F: Is f strictly monotonic? Use f ' (x)

$f'(x) = 3x^2 - 12x + 12$

$= 3(x^2 - 4x + 4)$

$= 3(x-2)^2 \geq 0$

Yes, f is strictly monotonic: f ' (x) $\geq$ 0 means that f(x) is increasing on whole domain

