## 4.2 Area

4.2 # 1-9, 33, definition of Area, p.260

Goals:
1. Understand the operation of summation and the symbol Σ
2. Understand the index of summation, usually indicated by an i or j
3. Learn the definition of Area and understand how it relates to limits of sums.



## 4.2 Area

Sigma Notation Σ

$\sum_{i=1}^{n}$ is a representation of **summation** from i = 1 to i = n

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + a_4 + \ldots + a_n$$

where ***i*** is the **index of summation**
and $a_i$ is the **ith term** of the sum



 

## 4.2 Area

$\sum_{a=1}^{n} a_i =$

---

$\sum_{y=3}^{5} \frac{1}{j} =$

---

$\sum_{k=0}^{4} \frac{1}{k^2+1} =$

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## 4.2 Area

$\sum_{a=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots + a_n$

---

$\sum_{y=3}^{5} \frac{1}{j} = \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$

---

$\sum_{k=0}^{4} \frac{1}{k^2+1} = \frac{1}{0+1} + \frac{1}{1+1} + \frac{1}{4+1} + \frac{1}{9+1} + \frac{1}{16+1}$

$\frac{1}{1} + \frac{1}{2} + \frac{1}{5} + \frac{1}{10} + \frac{1}{17}$

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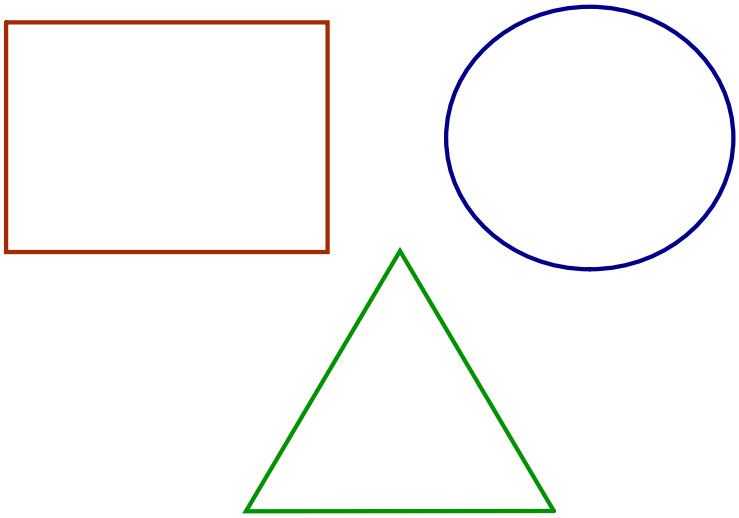
# 4.2 Area

How do we find the area of:



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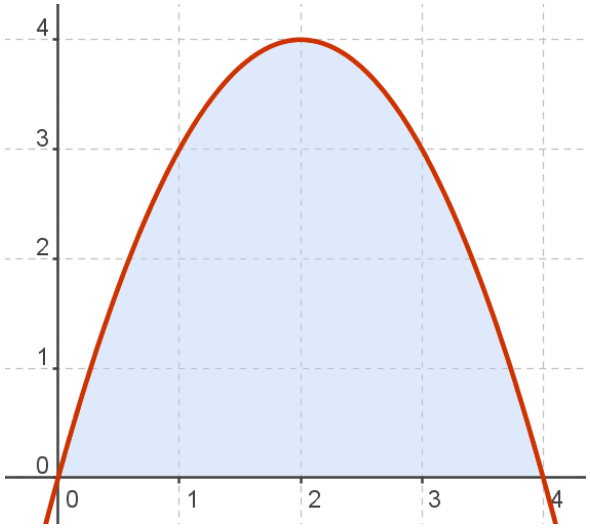
## 4.2 Area

What about the shaded area below?




Can we find the area with a formula
or other tools from algebra or geometry?

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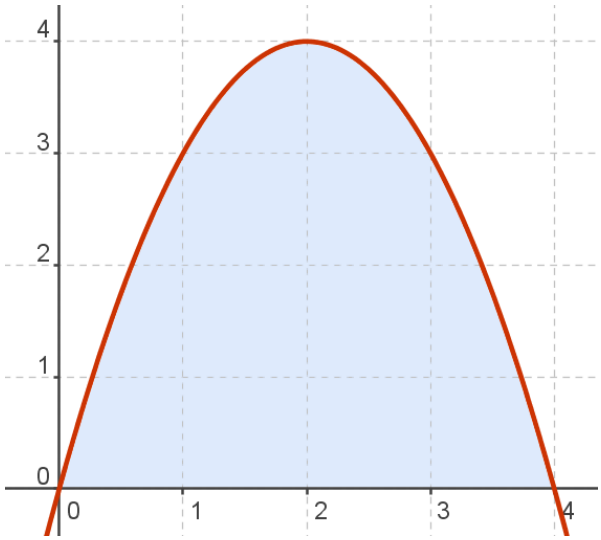
## 4.2 Area

How can we estimate the area?

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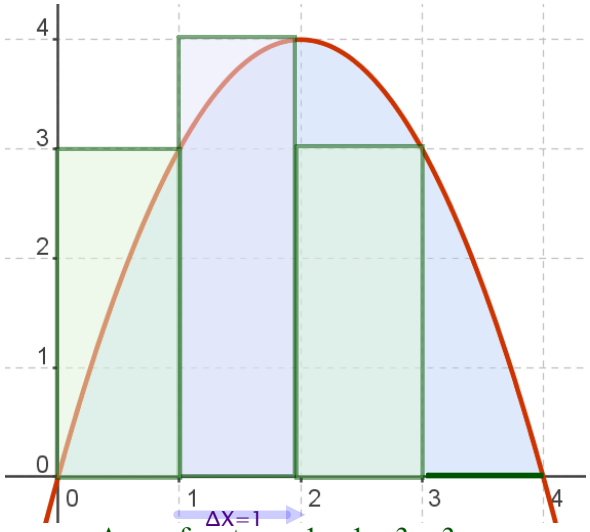
## 4.2 Area

Using 4 rectangles:




Area of rectange 1 = 1 x 3 = 3
Area of rectange 2 = 1 x 4 =4
Area of rectange 3 = 1 x 3 = 3
Area of rectange 4 = 1 x 0 = 0

$\Sigma = 10$

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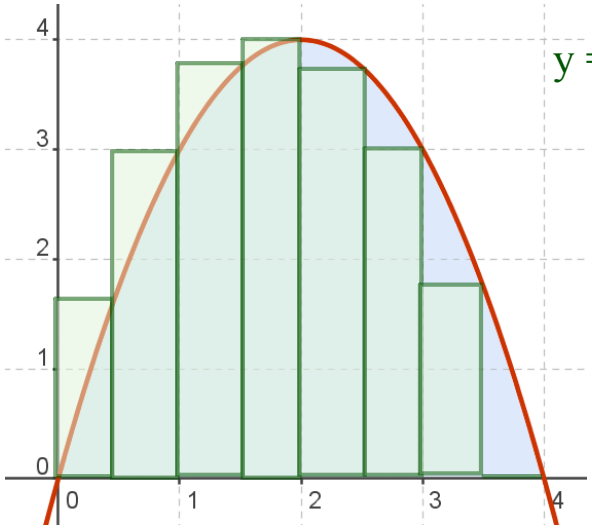
4.2 Area

Using 8 rectangles:




$$\sum_{i=1}^{8} \Delta x_i f(c_i)$$

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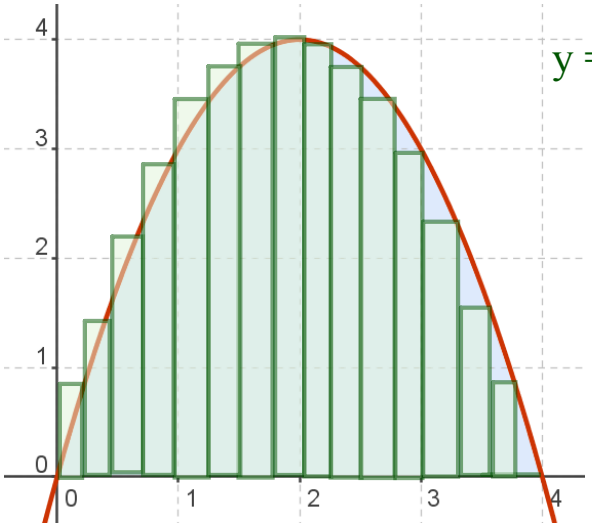
4.2 Area

Using 16 rectangles:




$$\sum_{i=1}^{16} \Delta x_i f(c_i)$$

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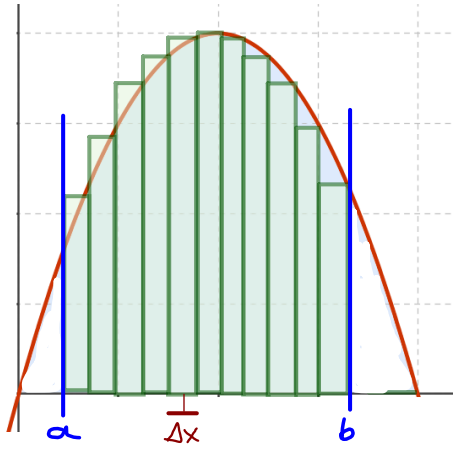
## 4.2 Area

To find the **area under the curve** over a specified interval, we would need to divide it into lots of little polygons, such as rectangles, for which we can find the area with known formulas.




$\Delta x = \frac{b-a}{n}$

where ***n*** is the number of rectangles

as $n \to \infty$
$\Delta x \to 0$

Areas of small rectangles

$\Delta x\, f(a)$ $\Delta x\, f(a+2\Delta x)$
$\Delta x\, f(a+\Delta x)$ $\Delta x\, f(a+3\Delta x)$

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## 4.2 Area

### Definition of Area of a Region in a Plane

Let f be continuous [1] and non-negative [2] on [a,b].
The area of the region bounded by the graph of f, the x-axis, and the vertical lines x = a and x = b is:

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x$$

$x_{i-1} < c_i < x_i$

$\Delta x = \frac{b-a}{n}$

as $n \to \infty$, $\Delta x \to 0$

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definition of area

## 4.2 Area

$$\sum_{k=5}^{8} k(k-4) =$$



## 4.2 Area

$$\sum_{k=5}^{8} k(k-4) = 5(1) + 6(2) + 7(3) + 8(4)$$
$$= 5 + 12 + 21 + 32$$
$$= 70$$



 

## 4.2 Area

$$\frac{9}{1+1}+\frac{9}{1+2}+\frac{9}{1+3}+\cdots+\frac{9}{1+14}$$



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## 4.2 Area

$$\frac{9}{1+1}+\frac{9}{1+2}+\frac{9}{1+3}+\cdots+\frac{9}{1+14}$$

$$\sum_{k=1}^{14}\frac{9}{1+k}$$

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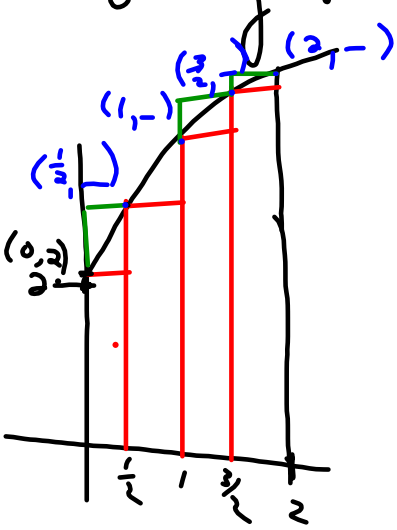
## 4.2 Area

G: $y = \sqrt{x} + 2$ F: upper & lower sums

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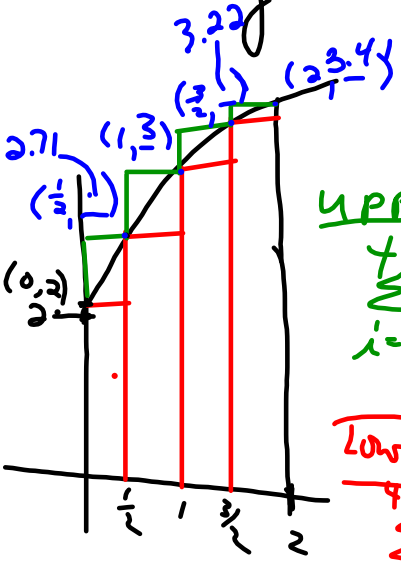
## 4.2 Area

G: $y = \sqrt{x} + 2$ F: upper & lower sums




$\Delta x = \frac{2-0}{4} = \frac{1}{2}$

Upper Sum.

$\sum_{i=1}^{4} \Delta x f(c_i) = \frac{1}{2}(2.71 + 3 + 3.22 + 3.41)$

$= 6.17$

Lower Sum

$\sum_{i=1}^{4} \Delta x f(c_i) = \frac{1}{2}(2 + 2.71 + 3 + 3.22)$

$= 5.47$

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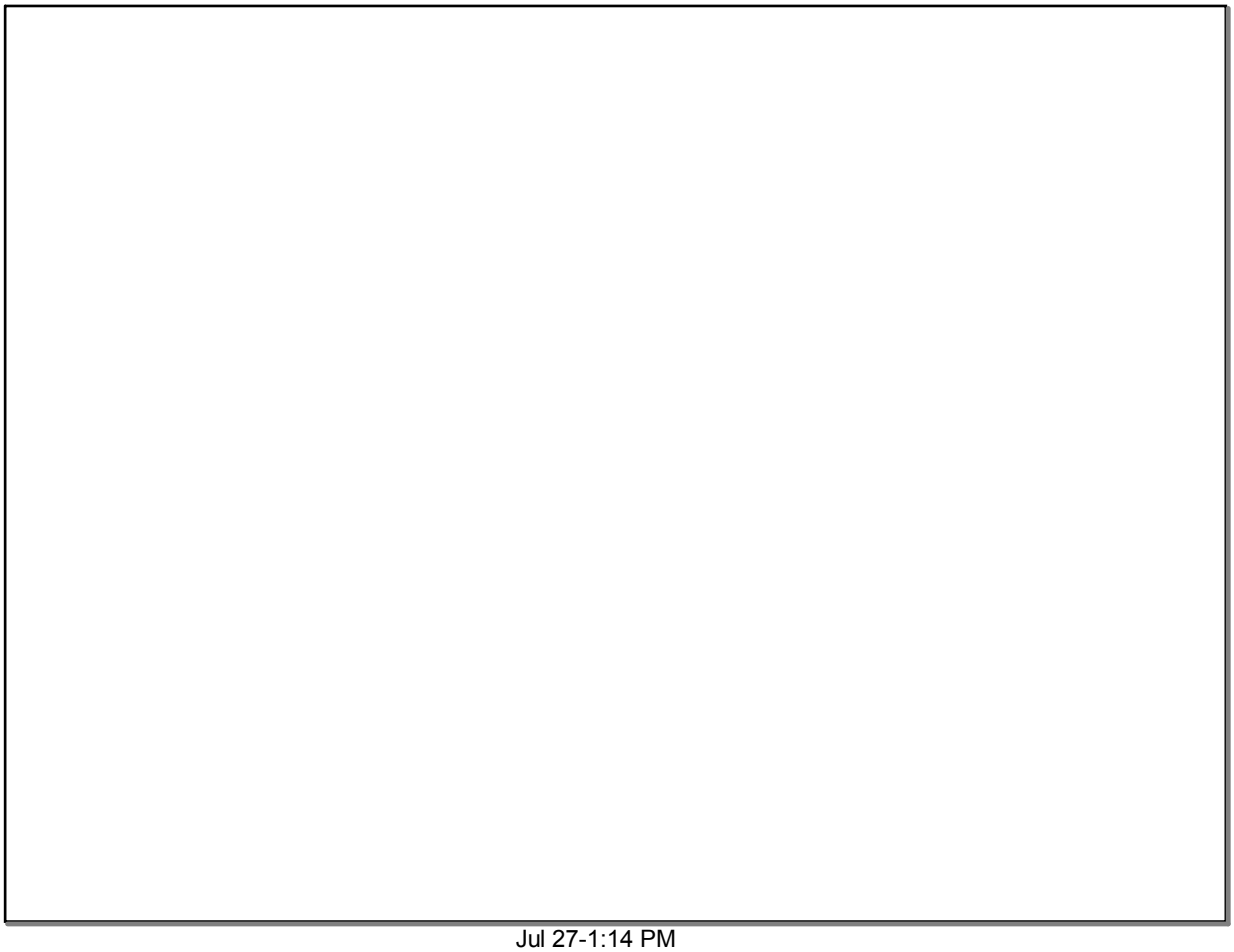

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