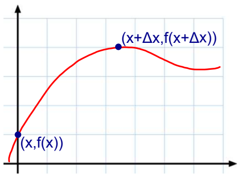
Quick Review of CALC 1 - Differential Calculus

Derivative of a Function

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$$

provided the limit exists



Notation: $f'(x)$, $\frac{dy}{dx}$

Can be interpreted as the slope of the tangent line to the curve at any point (x, f(x)) on the curve.

This generalizes from the Derivative at a specific point (c,f(c)) $\lim_{x \to c} \frac{f(x)-f(c)}{x-c}$ **to any point on the curve (x,f(x))**

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Quick Review of CALC 1 - Differential Calculus

Rules of Differentiation

1. The Constant Rule $\frac{d[c]}{dx} = 0$

***

2. The Power Rule $\frac{d[x^n]}{dx} = nx^{n-1}$

3. The Constant Multiple Rule $\frac{d[c f(x)]}{dx} = c f'(x)$

4. The Sum & Difference Rule

$$\frac{d[f(x) \pm g(x)]}{dx} = f'(x) \pm g'(x)$$

5. Trig Functions

$$\frac{d[\sin x]}{dx} = \cos x \qquad \frac{d[\cos x]}{dx} = -\sin x$$

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Quick Review of CALC 1 - Differential Calculus

co-functions

$\frac{d(\sin x)}{dx} = \cos x$

$\frac{d(\cos x)}{dx} = -\sin x$

$\frac{d(\tan x)}{dx} = \sec^2 x$

$\frac{d(\cot x)}{dx} = -\csc^2 x$

$\frac{d(\sec x)}{dx} = \sec x \tan x$

$\frac{d(\csc x)}{dx} = -\csc x \cot x$

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Quick Review of CALC 1 - Differential Calculus

Chain Rule

If:

1) $y = f(u)$ is a differentiable function of u, and

2) $u=g(x)$ is a differentiable function of x,

then $y = f(g(x))$ is a differentiable function of x, and

$$\frac{dy}{dx} = \frac{dy}{du} \bullet \frac{du}{dx}$$

or

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$

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co-functions

$\frac{d(\sin u)}{dx} = \cos u \frac{du}{dx}$ | $\frac{d(\cos u)}{dx} = -\sin u \frac{du}{dx}$

$\frac{d(\tan u)}{dx} = \sec^2 u \frac{du}{dx}$ | $\frac{d(\cot u)}{dx} = -\csc^2 u \frac{du}{dx}$

$\frac{d(\sec x)}{dx} = \sec u \tan u \frac{du}{dx}$ | $\frac{d(\csc x)}{dx} = -\csc u \cot u \frac{du}{dx}$

$\sin^2(3x^2-4x+1) + \cos^2(3x^2-4x+1) = 1$

More Rules of Differentiation

where u = f(x) and v = g(x)

1. The Product Rule $\frac{d[uv]}{dx} = uv' + vu'$

2. The Quotient Rule $\frac{d[u/v]}{dx} = \frac{vu' - uv'}{v^2}$

3. The Natural Logarithm $\frac{d[\ln u]}{dx} = \frac{u'}{u}$

4. The Exponential Function $\frac{d[e^u]}{dx} = e^u u'$

5. Inverse Trig Functions $\frac{d[\arcsin u]}{dx} = \frac{u'}{\sqrt{1-u^2}}$

$\frac{d[\arccos u]}{dx} = -\frac{u'}{\sqrt{1-u^2}}$ $\frac{d[\arctan u]}{dx} = \frac{u'}{1+u^2}$

Quick Review of CALC 1 - Differential Calculus

NOT calculus	vs	CALCULUS
Average Rate of Change on Interval		Instantaneous Rate of Change
Slope of Secant Line		Slope of Tangent Line
eg: $\frac{\text{total distance}}{\text{total time}}$		eg: instantaneous velocity

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Quick Review of CALC 1 - Differential Calculus

geogebra

Curve Sketching

Goal: Sketch graphs of functions using

1. Domain and range
2. Asymptotes
3. Relative Extrema
4. Concavity and Inflection Points.

Curve Sketching: DRIve A Car NEXt TRIP

D: Domain, R: Range, I: Intercepts, A: Asymptotes, CN: Critical Numbers, EXTR: Extrema, IP: Inflection Points

Other Applications

1. Max / Min: Optimization Problems
2. Related Rates

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Derivative Sketches

Homework

Quick Review of CALC 1 - **Integral** Calculus

Definition of Area of a Region in a Plane

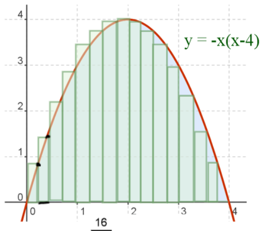
Let [1] f be continuous and [2] non-negative on [a,b].
The area of the region bounded by the graph of f, the x-axis, and the vertical lines x = a and x = b is:

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x$$

$x_{i-1} < c_i < x_i$ $\Delta x = \dfrac{b - a}{n}$

as $n \to \infty$, $\Delta x \to 0$

Using 16 rectangles:



$$\sum_{i=1}^{16} \Delta x_i f(c_i)$$

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Quick Review of CALC 1 - **Integral** Calculus

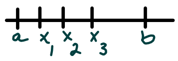
Definition of Riemann Sum

Let f be defined on [a,b], and let
Δ be a partition of [a,b], given by
$a = x_0 < x_1 < x_2 < \dots < x_{n-1} < x_n = b$
where Δx_i is the width of the ith sub-interval
If c_i is any point on the ith sub-interval, then the sum

$$\sum_{i=1}^{n} f(c_i)\,\Delta x_i \,, \qquad x_{i-1} < c_i < x_i$$

is called a Riemann Sum of f for the partition Δ

Δx_i not all equal



norm of the partition $\|\Delta\|$ =
width of longest subinterval

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x_i$$

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Quick Review of CALC 1 - **Integral** Calculus

Definition of Definite Integral

If f is defined on [a,b], and the following limit exists

$$\lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} f(c_i)\,\Delta x_i$$

Then f is integrable on [a,b] and the limit is denoted as:

$$\lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} f(c_i)\,\Delta x_i = \int_a^b f(x)\,dx$$

This is called the Definite Integral of f from a to b

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Quick Review of CALC 1 - **Integral** Calculus

Definition, **Antiderivative**:

A function F is an **antiderivative** of f on an interval **I** if $F'(x) = f(x)$ for all x in **I**.

A **Differential Equation** in x and y is an equation that involves x, y, and the derivative of y.

eg: $\frac{dy}{dx} = 5x^4 - 6x^2$

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Quick Review of CALC 1 - **Integral** Calculus

Fundamental Theorem of Calculus (FTC)

If : 1. a function ***f*** **is continuous on [a, b]** and

2. ***F*** **is an antiderivative of f** on the interval,

then:

$$\int_a^b f(x)dx = F(b)-F(a)$$

The integral of *f* from *a* to *b* is the difference:
(antiderivative of *f* evaluated at *x*=*b*) - (antiderivative of *f* evaluated at *x*=*a*.)

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Quick Review of CALC 1 - **Integral** Calculus

Fundamental Theorum of Calculus

If *f* is continuous on an open interval, *I*, containing ***a***,

then, for every ***x*** on the interval:

$$\frac{d}{dx}\left[\int_a^x f(t)dt\right] = f(x)$$

$$\frac{d}{dx}\left[\int_1^x t\,dt\right] = x$$

$$\frac{d}{dx}\left[\int_1^x \sqrt{t-1}\,dt\right] = \sqrt{x-1}$$

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Quick Review of CALC 1 - **Integral** Calculus

Fundamental Theorum of Calculus

If *f* is continuous on an open interval, *I*, containing ***a***, *and u = f(x)*

then, for every ***x*** on the interval:

$$\frac{d}{dx}\left[\int_a^u f(t)dt\right] = f(u)\,u'$$

$$\frac{d}{dx}\left[\int_1^{x^2} t\,dt\right] = x^2(2x) = 2x^3$$

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Quick Review of CALC 1 - **Integral** Calculus

Rules of:

Differentiation	Integration
$\frac{d(c)}{dx} = 0$	$\int 0\,dx = c$
$\frac{d(kx)}{dx} = k$	$\int k\,dx = kx + c$
$\frac{d(x^n)}{dx} = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + c$
$\frac{d(\sin x)}{dx} = \cos x$	$\int \cos x\,dx = \sin x + c$
$\frac{d(\cos x)}{dx} = -\sin x$	$\int \sin x\,dx = -\cos x + c$

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$-\int -\sin x\,dx = -\cos x + c$