# P.5 Factoring Polynomials

GOALS: Factor Polynomials using:

1. Distributive Property (common factors)
2. Trial and Error (trinomials)
3. Factor by Grouping (trinomials)

Study P.5 CVC 1-7, # 1, 5, 9, ....37, 39-55, 59, 65, 69, 73, ... 113



P.5 Factoring Polynomials

## Previous Multiplication

| Example 1 | Example 2 |
|---|---|
| $2x\,(x + y - xy)$ | $(2x + 1)(x - 3)$ |
| DP: $a(b + c) = ab + ac$ | F O I L |
| $2x(x) + 2x(y) - 2x(xy)$ | $2x(x) + 2x(-3) + 1x + 1(-3)$ |
| $2x^2 + 2xy - 2x^2y$ | $2x^2 - 5x - 3$ |



 

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Now: Reverse of Multiplication
**Find the Factored Form**

| □ ( ) | ( )( ) |
|---|---|
| | |
| $2x^2 + 2xy - 2x^2y$ | $2x^2 - 5x - 3$ |

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Now: Reverse of Multiplication
Find the Factored Form

**Always** look for common factor(s) first

Look for common factor(s)

$2x^2 + 2xy - 2x^2y$

Use the DP to factor

□ ( )

$a(b + c) = ab + ac$

$ab + ac = a(b + c)$

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## P.5 Factoring Polynomials

Now: Reverse of Multiplication
Find the Factored Form

$2x^2 + 2xy - 2x^2y$

Look for common factor(s)

$2x \cdot x + 2x \cdot y - 2x \cdot xy$

Use the DP to factor

$2x\,(x + y - xy)$

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## P.5 Factoring Polynomials

Factor: $32x^3y + 40x^2y^4$

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## P.5 Factoring Polynomials

Factor: $32x^3y + 40x^2y^4$

$8x^2y \cdot 4x + 8x^2y \cdot 5y^3$

$8x^2y(4x + 5y^3)$

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## P.5 Factoring Polynomials

Now: Reverse of Multiplication
**Find the Factored Form**

Note: No common factors
3 is prime

$2x^2 - 5x - 3$

Trinomial: Opening Ex.2

?

Product of 2 Binomials?

( )( )

Two Approaches:
1. Trial and Error
2. Factor by Grouping

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## P.5 Factoring Polynomials

### Trial and Error

Try

First

Last

$2x^2 - 5x - 3$

$(\quad)(\quad)$

Adjust until middle works

$(\square x \square)(\square x \square)$

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## P.5 Factoring Polynomials

### Trial and Error

Try

First

Last

$2x^2 - 5x - 3$

$(2x + 1)(x - 3)$

$x$

$-6x$

Adjust until middle works

$(2x + 1)(x - 3)$

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## Factor by Grouping: Overview

$ax^2 + bx + c$ or $ax^2 + bxy + cy^2$

example: $2x^2 + 7x + 3$

1. Put your trinomial into standard form, as above.
2. Want to find two integers:
   whose product is ac
   and whose sum is b

   ac = (2)(3)=6, so need
   ( ) ( ) = 6
   ( )+( ) = 7
   Use 1 and 6: $2x^2 + 1x + 6x + 3$
3. If you can find the two integers, replace bx with their sum.
   Then Factor by grouping.
4. If you cannot find the two integers, the trinomial is prime.

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$ax^2 + bx + c$

Factor by Grouping

$2x^2 - 5x - 3$

1. standard form:
2. a = __, **b = __**, c = __; **ac = __**
3. Find 2 integers: ( )( ) = ac, ( )+( )=b
   a) List factors of **ac**
   b) consider sums and differences
   c) determine signs
4. Substitute for **bx** getting two middle terms
5. Factor by grouping
6. Use Distributive Property to factor common binomial.

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Factor by Grouping

$ax^2 + bx + c$

1. standard form:
2. a = 2, b = -5, c = -3; ac = -6
3. Find 2 integers: ( )( ) = ac, ( )+( )=b
   a) List factors of **ac** *(use absoute value to start)*
   b) consider sums and differences
   c) determine signs
4. Substitute for **bx** getting two middle terms
5. Factor by grouping
6. Use Distributive Property to factor common binomial.

$2x^2 - 5x - 3$

$2x^2 - 5x - 3$

( )( ) = -6

( )+( ) = -5

$6 = 1 \cdot 6 \mid +7 \; -5$

*from difference need -5, so use -6 + 1*

$2x^2 - 5x - 3$

$2x^2 - 6x + x - 3$

$2x(x - 3) + (x - 3)$

$(2x + 1)(x - 3)$

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Factor by Grouping: **Detail**

$ax^2 + bx + c$ or $ax^2 + bxy + cy^2$

1. Put your trinomial into standard form, as above, and factor out all common factors.
2. Identify **a, b,** and **c** above.
3. Find two integers whose product is **ac** and whose sum is **b**.

   ( )( ) = **ac** and ( ) + ( ) = **b**

   HOW?

   a) List all possible factors of **ac**

   b) Compare the sum and differences of each pair of factors to **b**

   *If b is from a sum, then the numbers have the same sign*

   *If b is from a difference, then the numbers are opposite in sign*

   c) If no integers fit, then STOP. The trinomial is PRIME
3. Substitute for the **b**x term, using the sum of the two integers found above.
4. Factor by grouping: Factor common factors from:
   a) the 1st two terms only, then b) the 2nd two terms
5. Use the Distributive Property to factor the common binomial.

Check by multiplication

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## Factor by Grouping: Detail

$x^2 + 3x - 18$

1. In Standard form
2. a=1, b=3, c=-18, ac = -18
3. Find two integers whose product is **-18** and whose sum is **3**.
   ( )( ) = **ac** and ( ) + ( ) = **b**

a) List all possible factors of **18** (easiest to start with absolute value)
b) Compare the sum and differences of each pair of factors to **b**
c) **b = 3 and get 3** from **difference**
∴ integers have **opposite signs**;
since b is positive -> **use +6** and **-3**
results in (6)(-3) = -18 and 6 + (-3) = 3

| | Sum | Diff |
|---|---|---|
| 18 = 1(18) | 19 | 17 |
| = 2(9) | 11 | 7 |
| = 3(6) | 9 | 3 |

b=3

4. Substitute for the **bx** with **6x - 3x**
5. Factor by grouping: Common factors from:
   a) the 1st two terms only , then
   b) the 2nd two terms
6. Use the Distributive Property to factor the **common binomial, (x + 6)**.
7. Check by multiplication (not required)

$x^2 + 3x - 18$

$x^2 + 6x - 3x - 18$

$x(x + 6)$

$- 3(x + 6)$

$x(x + 6) - 3(x + 6)$

$(x - 3)(x + 6)$

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P.5 Factoring Polynomials

$ax^2+bx+c$

## Factor by Grouping

$x^2 - 8x + 15$

1. standard form:
2. a = **1** , **b = -8**, c = **15**; **ac = 15**
3. Find 2 integers: ( )( ) = ac, ( )+( )=b
   a) List factors of **ac** *(use absoute value to start)*
   b) consider sums and differences
   c) determine signs
4. Substitute for **bx** getting two middle terms
5. Factor by grouping
6. Use Distributive Property to factor common binomial.

| | + | - |
|---|---|---|
| 15 = | | |
| = | | |

$x^2 - 8x + 15$

$x^2 + (\quad)x + (\quad)x + 15$

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## P.5 Factoring Polynomials

$ax^2+bx+c$

### Factor by Grouping

$x^2 - 8x + 15$

1. standard form:
   a = 1, **b = -8**, c = **15**; **ac = 15**
2. Find 2 integers: mn = ac, m+n=b
   a) List factors of **ac**
   b) consider sums and differences
   c) determine signs
3. Substitute for **bx** getting two middle terms
4. Factor by grouping
5. Use Distributive Property to factor common binomial.

| | + | - |
|---|---|---|
| 15 = 1(15) | 16 | 14 |
| = 3 (5) | 8 | 2 |

Get 8 from sum
∴ Same sign.
Need -8, so use -3 and -5

$x^2 - 8x + 15$

$x^2 +(-3)x +(-5)x + 15$

$(x)(x-3)+(-5)(x-3)$

$(x-5)(x-3)$

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## P.5 Factoring Polynomials

$ax^2+bx+c$

### Factor by Grouping

$7x^2 + 21X + 14$

1. standard form:
2. a = __, **b = __**, c = __; **ac = __**
3. Find 2 integers: ( )( ) = ac, ( )+( )=b
   a) List factors of **ac**
   b) consider sums and differences
   c) determine signs
4. Substitute for **bx** getting two middle terms
5. Factor by grouping
6. Use Distributive Property to factor common binomial.

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## P.5 Factoring Polynomials

$7x^2 + 21x + 14$

$7(x^2 + 3x + 2)$

( )( ) = 2
( )+( ) = 3

(2)(1) = 2
(2)+(1) = 3

Factor by Grouping, +

1. standard form:
2. a = __, **b =** __, c = __; **ac =** __ .
3. Find 2 integers: ( )( ) = ac, ( )+( )=b
   a) List factors of **ac**
   b) consider sums and differences
   c) determine signs
4. Substitute for **bx** getting two middle terms
5. Factor by grouping
6. Use Distributive Property to factor common binomial.

$3x$

$(7)[x^2 + (2)x + (1)x + 2]$ group

$(7)[(x)(x+2)+(1)(x+2)]$

$(7)(x+1)(x+2)$

Why are [ ] needed above, but not at end?

Above is 7 times a sum.
End is product of factors.



## P.5 Factoring Polynomials

$ax^2 + bx + c$

$6x^2 + 9x - 60$

1. standard form:
   a = __, **b =** __, c = __; **ac =** __
2. Find 2 integers: mn = ac, m+n=b
   a) List factors of **ac**
   b) consider sums and differences
   c) determine signs
3. Substitute for **bx** getting two middle terms
4. Factor by grouping
5. Use Distributive Property to factor common binomial.

## P.5 Factoring Polynomials

$ax^2+bx+c$

$6x^2+9x-60$

$3(2x^2+3x-20)$

$ac=-40$

| $40=$ | + | − |
|---|---|---|
| $1\cdot 40$ | 41 | 39 |
| $2\cdot 20$ | 22 | 18 |
| $4\cdot 10$ | 14 | 6 |
| $5\cdot 8$ | 13 | 3 |

$(8)(-5)=-40$

$(8)+(-5)=3$

$3(2x^2+8x-5x-20)$

$3[2x(x+4)-5(x+4)]$

$3(2x-5)(x+4)$

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## P.5 Factoring Polynomials

$12w^2-19w-10$

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## P.5 Factoring Polynomials

$12w^2 - 19w - 10$

$ac = -120$

$b = -19$

$(5)(-24) = -120$

$(-24) + (5) = -19$

$12w^2 - 24w + 5w - 10$

$12w(w-2) + 5(w-2)$

$(12w+5)(w-2)$

Foil

| | + | − |
|---|---|---|
| $120 = 1 \cdot 120$ | 121 | 119 |
| $2 \cdot 60$ | 62 | 58 |
| $3 \cdot 40$ | 43 | 37 |
| $5 \cdot 24$ | 29 | 19 |
| $6 \cdot 20$ | | |
| $8 \cdot 15$ | | |
| $4 \cdot 30$ | | |
| $10 \cdot 12$ | | |



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$12w^2 - 20w - 10$

## P.5 Factoring Polynomials

$12w^2 - 20w - 10$

$2(6w^2 - 10w - 5)$

prime

( )( ) = -30

( ) + ( ) = -10

| | + | − |
|---|---|---|
| $30 = 1 \cdot 30$ | 31 | 29 |
| $= 2 \cdot 15$ | 17 | 13 |
| $= 3 \cdot 10$ | 13 | 7 |
| $= 5 \cdot 6$ | 11 | 1 |

$w^2 - 10$

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## P.5 Factoring Polynomials

$ax^2 + bx + c$

$x^2 + 3x - 18$

$a = 1$
$b = 3$
$c = -18$

$x^2 + 4x - 18$

$a = 1$
$b = 4$
$c = -18$

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## P.5 Factoring Polynomials

$ax^2+bx+c$

$x^2 + 3x - 18$

$a=1$
$b=3$
$c=-18$

$ac=-18$
$b=3$

| | + | − |
|---|---|---|
| $18=1\cdot18$ | 19 | 17 |
| $=2\cdot9$ | 11 | 7 |
| $=3\cdot6$ | 9 | 3 |

$(-6)(3)=-18$
$(+6)+(-3)=3$

$x^2+6x-3x-18$ group

$x(x+6)-3(x+6)$

$(x-3)(x+6)$

$x^2+4x-18$

NO

$a=1$
$b=4$
$c=-18$

∴ prime



## P.5 Factoring Polynomials

$ax^2+bx+c$

$3x-6+30x^2$

## P.5 Factoring Polynomials

$3x-6+30x^2$

$30x^2+3x-6$

$3(10x^2+x-2)$

$3[10x^2-4x+5x-2]$

$3[2x(5x-2)+1(5x-2)]$

$3(2x+1)(5x-2)$

$ax^2+bx+c$

$ac=10(-2)=-20$

$(-4)(5)=-20$

$(-4)+(5)=1$

$20=1\cdot 20$ 19

$2\cdot 10$ 8

$4\cdot 5$ 1

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$8x^2+37x-15$

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## P.5 Factoring Polynomials

$8x^2+37x-15$

ac = 8(-15)= -120
b = 37

| | + | − |
|---|---|---|
| 120 = 1·120 | 121 | 119 |
| 2·60 | 62 | 58 |
| 3·40 | 43 | 37 |

$(-3)(40)=-120$

$(-3)+(40)=37$

$8x^2-3x+40x-15$

$x(8x-3)+5(8x-3)$

$(x+5)(8x-3)$

check by multiplying:

FOIL $(8x^2-3x+40x-15)$
$(8x^2+37x-15)$

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