1.4 Complex Numbers

GOALS:

1. Define complex numbers as those that include negative radicals.
2. Define $i = \sqrt{-1}$ and $i^2 = -1$
3. Perform operation of addition, subtraction, multiplication and division with complex numbers.

Study 1.4 CVC 1-7

1, 5, 9, 13, ... 57; 55

1.4 Complex Numbers

Introduction: Previously...

$\sqrt{25} = \_\_\_$

$\sqrt{18} = \sqrt{\_\_}\,\sqrt{\_\_} = \_\sqrt{\_\_}$

$\sqrt{8} = \sqrt{\_\_}\,\sqrt{\_\_} = \_\sqrt{\_\_}$

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1.4 Complex Numbers

Introduction: Previously...

$\sqrt{25} = 5$

$\sqrt{18} = \sqrt{9}\sqrt{2} = 3\sqrt{2}$

$\sqrt{8} = \sqrt{4}\sqrt{2} = 2\sqrt{2}$

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1.4 Complex Numbers

Introduction: What about...?

$\sqrt{-25} = ?$ **not real**

$\sqrt{-25} = \sqrt{}\sqrt{-1} = \sqrt{-1}$

Can we define $\sqrt{-1}$

so we can use it in computations?

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1.4 Complex Numbers

Introduction: What about...?

$\sqrt{-25}$ = ? not real

$\sqrt{-25} = \sqrt{25}\,\sqrt{-1} = 5\sqrt{-1}$

Can we define $\sqrt{-1}$ so we can use it in computations?

Yes! Define:

$i = \sqrt{-1}$ and $i^2 = -1$

$i^2 = [(-1)^{1/2}]^2 = (-1)^1 = -1$

power rule

1.4 Complex Numbers

$i = \sqrt{-1}$ and $i^2 = -1$

Define: Complex Number

$a + bi$ eg: $3 + 2i$

where a is real & bi is imaginary

If $a = 0$, then $0 + bi = bi$ eg: $2i$

pure imaginary

If $b = 0$, then $a + 0i = a$ eg: 3

real number

1.4 Complex Numbers

$i = \sqrt{-1}$ and $i^2 = -1$

Operations with Complex Numbers

$a + bi$

treat bi terms as if it were a variable term and complete the operation & simplification

Note: i is not a variable

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1.4 Complex Numbers

$i = \sqrt{-1}$ and $i^2 = -1$

Addition:

$(-2 + 6i) + (4 - i)$

Operations with Complex Numbers

$a + bi$

treat bi terms as if it were a variable term and complete the operation & simplification

Note: i is not a variable

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1.4 Complex Numbers

$i = \sqrt{-1}$ and $i^2 = -1$

Operations with Complex Numbers
a + bi

treat *bi* terms as if it were a variable term
and complete the operation & simplification

Note: *i* is not a variable

Addition:

$$(-2 + 6i) + (4 - i)$$

$$-2 + 6i + 4 - i$$

$$-2 + 4 + 6i - i$$

$$2 + 5i$$

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1.4 Complex Numbers

$i = \sqrt{-1}$ and $i^2 = -1$

Operations with Complex Numbers
a + bi

treat *bi* terms as if it were a variable term
and complete the operation & simplification

Note: *i* is not a variable

Subtraction:

$$(-2 + 6i) - (4 - i)$$

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1.4 Complex Numbers

$i = \sqrt{-1}$ and $i^2 = -1$

Operations with Complex Numbers
a + bi

treat *bi* terms as if it were a variable term
and complete the operation & simplification

Note: *i* is not a variable

Subtraction:

$(-2 + 6i) - (4 - i)$

$-2 + 6i - 4 + i$

$-6 + 7i$

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1.4 Complex Numbers

$i = \sqrt{-1}$ and $i^2 = -1$

Operations with Complex Numbers
a + bi

treat *bi* terms as if it were a variable term
and complete the operation & simplification

Note: *i* is not a variable

Multiplication:

$(-2 + 6i)(4 - i)$

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1.4 Complex Numbers

$i = \sqrt{-1}$ and $i^2 = -1$

Operations with Complex Numbers
$a + bi$

treat bi terms as if it were a variable term and complete the operation & simplification

Note: i is not a variable

Multiplication:

$(-2 + 6i)(4 - i)$

FOIL: $(-2)(4) + (-2)(-i) + (6i)(4) + (6i)(-i)$

$(-8) + (2i) + (24i) + (-6i^2)$

$-8 + 26i + (-6)(-1)$

$-8 + 26i + 6$

$-2 + 26i$

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1.4 Complex Numbers

$i = \sqrt{-1}$ and $i^2 = -1$

Division:

$$\frac{(-2 + 6i)}{(4 - i)}$$

How?

Use Multiplication Property of One:

1. Want result in a + bi form with no denominator.
2. Use process similar to rationalizing the denominator. In this case, want a denominator that is a real number (no complex part).

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1.4 Complex Numbers

$i = \sqrt{-1}$ and $i^2 = -1$

How?

Division:

$$\frac{(-2 + 6i)}{(4 - i)}$$

$$\frac{(-2 + 6i)}{(4 - i)} \cdot \frac{?}{(4 + i)}$$

Use
Multiplication Property of One

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1.4 Complex Numbers

$i = \sqrt{-1}$ and $i^2 = -1$

How?

Division:

$$\frac{(-2 + 6i)}{(4 - i)}$$

$$\frac{(-2 + 6i)}{(4 - i)} \cdot \frac{(4 + i)}{(4 + i)}$$

Use
Multiplication Property of One

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1.4 Complex Numbers

$i = \sqrt{-1}$ and $i^2 = -1$

How?

Division:

$$\frac{(-2 + 6i)}{(4 - i)}$$

$$\frac{(-2 + 6i)}{(4 - i)} \cdot \frac{(4 + i)}{(4 + i)}$$

$$\frac{(-2 + 6i)(4 + i)}{(4 - i)(4 + i)} = \frac{-8 - 2i + 24i + 6i^2}{16 - i^2}$$

$$= \frac{-8 + 22i + 6(-1)}{16 - (-1)} = \frac{-14 + 22i}{17}$$

$$= \frac{-14}{17} + \frac{22}{17} i$$

a + b*i* form

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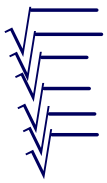
1.4 Complex Numbers

Simplify:

$i = \sqrt{-1}$ and $i^2 = -1$

1. $7 - (-9 + 2i) - (-17 - i)$
2. $(2 + 7i)(2 - 7i)$
3. $\frac{5i}{2 - i}$
4. $5\sqrt{-8} + 3\sqrt{-18}$
5. $(-5 - \sqrt{-9})^2$
6. $\sqrt{-12}\,(\sqrt{-4} - \sqrt{2})$
7. $\frac{-15 - \sqrt{-18}}{33}$



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1.4 Complex Numbers

Simplify:

$i = \sqrt{-1}$ and $i^2 = -1$

1. $7 - (-9 + 2i) - (-17 - i)$ $\quad 33 - i$
2. $(2 + 7i)(2 - 7i)$ $\quad 53$
3. $\dfrac{5i}{2 - i}$ $\quad -1+2i$
4. $5\sqrt{-8} + 3\sqrt{-18}$ $\quad 19\, i\sqrt{2}$
5. $(-5 - \sqrt{-9})^2$ $\quad 16 + 30\, i$
6. $\sqrt{-12}\,(\sqrt{-4} - \sqrt{2})$ $\quad -4\sqrt{3} - 2\, i\sqrt{6}$
7. $\dfrac{-15 - \sqrt{-18}}{33}$ $\quad \dfrac{-5}{11} - \dfrac{i\sqrt{2}}{11}$

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1. $7 - (-9 + 2i) - (-17 - i)$ $\quad 33 - i$

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1. $7 - (-9 + 2i) - (-17 - i)$ $33 - i$

$7 + 9 - 2i + 17 + i$

$33 - i$

$-(a + b)$

$-a - b$

3. $\dfrac{5i}{2 - i}$ $-1+2i$

3. $\dfrac{5i}{2-i}$

-1+2i

$\dfrac{5i(2+i)}{(2-i)(2+i)} = \dfrac{10i+5i^2}{4-i^2}$

$i^2 = -1$

$= \dfrac{10i+5(-1)}{4-(-1)}$

$ii = \sqrt{-1}\sqrt{-1}$

$= (\sqrt{-1})^2$

$= -1$

$= \dfrac{-5+10i}{5} = \dfrac{5(-1+2i)}{5_1}$

$= -1+2i$

5. $(-5-\sqrt{-9})^2$

5. $(-5 - \sqrt{-9})^2$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a+b)(a+b)$

$(-5-3i)^2$

$(-5)^2 + 2(-5)(-3i) + (-3i)^2$

$25 + 30i + 9i^2$

$i^2 = -1$

$25 + 30i + 9(-1)$

$16 + 30i$

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6. $\sqrt{-12}\,(\sqrt{-4} - \sqrt{2})$

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6. $\sqrt{-12}\,(\sqrt{-4} - \sqrt{2})$

$\sqrt{-12} = \sqrt{4}\sqrt{3}\sqrt{-1}$
$= 2i\sqrt{3}$

$2i\sqrt{3}\,(2i - \sqrt{2})$

$4i^2\sqrt{3} - 2i\sqrt{3}\sqrt{2}$

$4(-1)\sqrt{3} - 2i\sqrt{6}$

$-4\sqrt{3} - 2i\sqrt{6}$

7. $\dfrac{-15 - \sqrt{-18}}{33}$

7. $\dfrac{-15-\sqrt{-18}}{33}$

$\sqrt{-18} = \sqrt{9}\sqrt{2}\sqrt{-1}$

$3 \quad i \quad \sqrt{2}$

$\dfrac{-15-3i\sqrt{2}}{33}$

$a(b+c) = ab + ac$

$\dfrac{3(-5)+3(-i\sqrt{2})}{33}$

$\dfrac{3(-5-i\sqrt{2})}{33} = \dfrac{-5-i\sqrt{2}}{11} = -\dfrac{5}{11} - \dfrac{i\sqrt{2}}{11}$

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